# THE EFFECTS OF A STRUCTURED CURRICULUM ON THE LEARNING OF MULTIPLICATION FACTS FOR MAINSTREAMED ELEMENTARY STUDENTS WITH DISABILITIES

BY

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by

Carolyn Anne Harris

This work is dedicated to
Cecil Mercer and Susan Miller
who shared what they knew
and to all of the teachers and students
who may benefit from that knowledge.

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By

Carolyn Anne Harris

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Educational placement decisions often make regular classroom teachers responsible for providing the individualized instruction required by special education students. Appropriate and successful learning experiences are possible through teachers' consistent use of procedures associated with successful student outcomes. Effective instructional procedures have been identified for general academic learning and extended to the area of mathematics.

The basal curriculum materials typically used for elementary mathematics instruction contain few of the best teaching practices identified for mathematics. Curriculum materials are needed that incorporate the features of effective instruction and increase mainstreamed students' chances for success in general education settings.

This investigation examined the effects of using a research-based curriculum to teach multiplication to large groups of students with varying ability and skill levels in mainstream elementary classrooms. The curriculum has been field tested for students with learning problems in special education settings. This study extended the results of field testing to students with disabilities who receive mathematics instruction in regular classroom settings.

Three experimental questions were asked about the effects of instruction on students' understanding of multiplication, computational fluency, and performance levels within a large group. Two studies were conducted using single subject methodology to evaluate the effects of instruction and establish experimental control.

Subjects included 13 students with disabilities and their normally achieving classmates in six second grade classrooms. Instructional groups ranged in size from 25 to 27 students. Teachers followed scripted procedures to provide systematic and explicit instruction in the concepts of multiplication. Instruction also included fluency practice and generalization to complex word problems. Performance was monitored on untimed practice problems included with each lesson as well as on timed probes of multiplication facts.

The findings of the investigation showed that students with disabilities learned to solve multiplication problems accurately and increased their rates of computation during large-group instruction under regular classroom conditions with use of the structured curriculum. Similar performance profiles for students with disabilities and their normally achieving classmates confirmed the effectiveness of the curriculum as a mainstream intervention. The results of this investigation have implications for classroom practice, curriculum planning, and future research.

# CHAPTER I
# INTRODUCTION

Many of the instructional and curricular traditions of American education are founded on the "bizarre assumption that students are to be uniformly ready for whatever custom dictates that they are to be ready for--from the day of their arrival until the day of their departure from the system" (Goodlad, 1990, p. 23). Slavin (1990) used the phrase "one-size-fits-all instruction" (p. 40) to describe the standard curriculum presented "at the same rate and in the same way to groups of students" (p. 40) in classrooms throughout the country. Students who can benefit from such instruction learn at an acceptable rate. Other students, however, such as those with disabilities, often have difficulty learning "at the same rate or with the same instructional materials as the majority of their peers" (Trent, 1989, p. 24) and are soon academically distanced from their normally achieving classmates.

Two approaches are frequently used to help students with disabilities. In one well-established approach, students with disabilities are removed from mainstream classrooms and placed in self-contained special education settings where they receive individualized instruction in one or more academic areas from specialist teachers (Haring, 1986; Jenkins & Pious, 1991; Ysseldyke, 1989). Students with mild to moderate learning problems often join their normally achieving peers for social, nonacademic, and selected academic classroom activities.

In another placement option, known as mainstreaming, students with a wide range of learning problems participate more fully in regular classroom life. Although mainstreamed students are identified as needing specialized educational services in academic areas, they remain in classroom settings with normally achieving peers. Regular classroom teachers are expected to adapt instruction to meet the specific learning needs of their students with disabilities. Depending on the resources available, teachers receive varying degrees of preparation, consultation, and support for teaching mainstreamed students.

Although a desirable social arrangement, mainstreaming has not been particularly effective academically (Carnine & Kameenui, 1990; Fuchs & Fuchs, 1988; Hallahan, Keller, McKinney, Lloyd, & Bryan, 1988; Kauffman, Gerber, & Semmel, 1988; Schumaker & Deshler, 1988). In fact, recent investigations (Baker & Zigmond, 1990; Semmel, Abernathy, Butera, & Lesar, 1991; Silver, 1991; Simmons, Fuchs, & Fuchs, 1991; Ysseldyke, Thurlow, Wotruba, & Nania, 1990) reveal the disturbing reality that classroom teachers often continue with "business as usual" (Zigmond & Baker, 1990, p. 185) when students with disabilities are placed in their classrooms.

Without improvement of instructional conditions in general education, mainstreaming efforts will continue to fail. The gap between students who have difficulty learning and their normally achieving peers will remain. Fortunately, improvement is possible through implementation of instructional procedures that have been associated with successful student outcomes (Gersten & Woodward, 1990; Kameenui & Simmons, 1990; Keogh, 1990; Leinhardt & Pallay, 1982; Nowacek, McKinney, & Hallahan, 1990; Slavin, 1990).

## Effective Teaching

Important contributions to the knowledge base on understanding and improving human learning have been made within diverse theoretical paradigms including cognitive and behavioral learning theories. From various theoretical perspectives, effective teaching researchers have sought to identify the classroom conditions and teaching behaviors consistently associated with student success in both regular and special education settings (Brophy, 1986;

Brophy & Good, 1986; Christenson, Ysseldyke, & Thurlow, 1989; Kameenui & Simmons, 1990; Larrivee, 1986; Leinhardt & Pallay, 1982; Mastropieri, Scruggs, & Shiah, 1991; Rosenshine & Stevens, 1986).

Regardless of the theoretical foundation, consistent use of the practices identified by effective teaching researchers would enable classroom teachers to provide students with more successful learning opportunities through instruction based on individual learning needs. As a result, the inevitability of academic failure could be avoided for many students who have difficulty learning in traditional classroom environments. Unfortunately, efforts to operationalize the effective teaching model for general classroom application have resulted in a veritable "laundry list of factors related to student achievement" (Christenson et al., 1989, p. 21).

Mercer (1992) reviewed the research on effective teaching and integrated various lists of recommended behaviors. In the resulting synthesis, Mercer (C. D. Mercer, personal communication, December 1, 1991) organized critical teaching functions according to the expected level of impact on student performance (Table 1). For general academic learning, major impact can be expected from (a) using direct instruction, (b) providing opportunities for

Table 1

Organizing Framework for Effective Teaching Behaviors

| Major Impact | Substantial Impact | Important Impact |
|---|---|---|
| • Use direct instruction | • Focus on time for learning | • Monitor progress frequently |
| • Provide success | | |
| • Give appropriate feedback | • Establish goals and expectancies | • Provide a positive environment |
| • Teach students to generalize | • Teach independent learning | • Use effective management |

success, (c) giving appropriate feedback, and (d) teaching students to generalize. Substantial impact can be realized from (a) focusing on time for learning, (b) establishing goals and expectancies, and (c) teaching independent learning. Important impact can be achieved by (a) monitoring progress, (b) providing a positive environment, and (c) using effective management.

The instructional behaviors described by Mercer (C. D. Mercer, personal communication, December 1, 1991) offer teachers a way to improve learning opportunities for even their most difficult-to-teach students. While the principles of effective teaching can improve conditions in all academic areas, they appear particularly relevant to mathematics instruction at the elementary level.

## Mathematics Instruction

Mathematics is widely recognized as a source of academic difficulty for many students (Bartel, 1990; Cawley & Miller, 1989; Christensen & Gerber, 1990; Fletcher & Loveland, 1986; Katsiyannis & Prillaman, 1990; Kirby & Becker, 1988; Lessen, Dudzinski, Karsh, & Van Acker, 1989; Silbert, Carnine, & Stein, 1990; Stein, Jenkins, & Arter, 1983). Low performance levels are found at all grade levels throughout the United States (Anrig & LaPointe, 1989; Silver et al., 1988).

Mercer (1992) and others (Baroody & Hume, 1991; Cawley, Fitzmaurice-Hayes, & Shaw, 1988; Cawley & Parmar, 1990) identify poor instruction as a contributing factor to the mathematical difficulties experienced by many students. Recognizing that traditional methods of instruction are seriously inadequate, the National Council of Teachers of Mathematics has called for revision at the national level, and standards have been developed to guide reform efforts (NCTM, 1988, 1989).

The importance of problem solving is emphasized throughout the NCTM standards (NCTM, 1989). Computational proficiency, a basic tool for problem solving (Goldman, 1989; Hasselbring, Goin, & Bransford, 1987; Silbert et al., 1990), is addressed in the recommendations for early instruction. According to the standards for kindergarten through fourth grade, instruction in whole number operations should include both conceptual and procedural knowledge. Prior to symbolic tasks, students should have concrete experiences that lead to conceptual knowledge through reasoning and critical thinking. Conceptual knowledge should then be linked to the language, uses, symbolism, and procedures of computation.

For mastery of computational facts, developers of the NCTM standards recommend instruction that includes mathematical concepts; concrete experiences; activities that link manipulative experiences to procedural steps; and strategies for mental computation, estimation, and checking results. The outcome of such instruction should be "good achievement, good retention, and a reduction in the amount of time children need to master computational skills" (NCTM, 1989, p. 44).

The NCTM standards provide general guidelines for reforming mathematics instruction in the United States. In the standards related to computation (NCTM, 1989), for example, teachers are directed to "develop meaning for the operations" (p. 41) and to help students develop "thinking strategies for learning basic facts" (p. 44). Examples are described, but specific teaching procedures are not provided. How is meaning for each operation developed? How do students learn strategies for computing basic facts accurately and fluently? Before the standards can actually influence classroom practices, their implementation must be made explicit through identification of specific teaching procedures that are effective for mathematical learning.

Based on an extensive review of cognitive and behavioral learning theories as well as theories related to curriculum design, Mercer and Miller (1992c) concluded that mathematics can be taught effectively through a blending of cognitive and behavioral principles. Their review enabled Mercer and Miller (1992c) to identify specific components of mathematics instruction that fit into the framework proposed by Mercer (C. D. Mercer, personal communication, December 1, 1991). Components include (a) systematic and explicit instruction, (b) development of concepts, (c) feedback, (d) generalization training, (e) practice to mastery levels, (f) clear goals and expectations, (g) problem-solving instruction, (h) continuous monitoring, (i) attention to attitudes about mathematics, and (j) selection of appropriate content (Table 2).

In general, the basal curricula that provide the foundation for elementary mathematics instruction do little to facilitate teachers' use of best practices (Darch, Carnine, & Gersten, 1984; Engelmann, Carnine, & Steely,

Table 2
Critical Components of Effective Mathematics Instruction

| Effective Teaching Behaviors For General Academic Learning (C. D. Mercer, personal communication, December 1, 1991) | | Components of Effective Mathematics Instruction (Mercer & Miller, 1992c) |
|---|---|---|
| Use direct instruction | --> | Provide systematic and explicit instruction |
| Provide success | --> | Teach students to understand mathematical concepts |
| Give appropriate feedback | --> | Provide feedback |
| Teach students to generalize | --> | Teach generalization |
| Focus on time for learning | --> | Teach to mastery levels |
| Set goals and expectancies | --> | Set goals and expectancies |
| Teach independent learning | --> | Teach problem solving |
| Monitor progress | --> | Monitor progress |
| Provide a positive environment | --> | Promote a positive attitude toward mathematics |
| Use effective management | --> | Select appropriate content |

1991; Kameenui, Carnine, Darch, & Stein, 1986; Silbert & Carnine, 1991; Woodward, 1991). Few features of effective instruction are found in even recent editions of basal series (Silbert & Carnine, 1990). Carnine (1990) cautioned that students' performance in mathematics cannot improve without reform of the basal textbooks that "define the curriculum" (p. 15).

## Basal Instruction in Mathematics

The typical basal mathematics curriculum, commonly used for whole-class instruction at the elementary level, follows a developmental, spiraling approach to instruction. In a typical basal series (Rucker & Dilley, 1979), elementary students are expected to become proficient in the basic skills of counting and place value; addition, subtraction, multiplication, and division of whole numbers; fractions; decimals; estimation; geometry; measurement; problem solving; graphing; money; and time. Skills are introduced rapidly in topical units and are repeated at more difficult skill levels throughout the grades. Teacher demonstration of new skills is followed by independent practice with abstract materials. Students move through the curriculum by completing unit worksheets and tests.

Critics of basal instruction describe several features that are inconsistent with the elements of effective instruction. Teaching manuals offer general suggestions instead of explicit teaching formats for direct instruction (Carnine, 1990). Rapid introduction of new skills and cursory coverage lead to confusion and failure for many students (Silbert & Carnine, 1990; Woodward, 1991). Practice is fixed in frequency and confined to a narrow range of examples that limits opportunities for generalization (Cawley & Parmar, 1990). Teachers may be directed to urge students to work quickly, but actual fluency practice and mastery criteria are typically omitted (Silbert et al., 1990). Objectives often refer to nonspecific skill hierarchies (Kameenui & Simmons, 1990).

Basal strategy instruction generally lacks the explicit demonstration, modeling, practice, and systematic review required for students to use solution strategies independently (Engelmann et al., 1991). Progress is infrequently monitored and rarely charted or used for guiding instruction. "Handicapped by meaningless instruction" (Baroody & Hume, 1991, p. 54), many students develop lifelong negative attitudes toward mathematics.

Spiraling basal instruction, designed to add depth, actually results in "superficial coverage of a large number of topics each year" (Carnine, 1990, p. 5). Without systematic instruction, adequate practice, and cumulative review during the elementary years, many students fail to master the basic concepts and skills that are essential for success at higher levels. Mathematics becomes just "an array of unconnected and incomprehensible facts and procedures--strange stuff they are supposed to memorize" (Baroody, 1989, p. 1).

Once teachers become aware of the deficiencies of traditional procedures, "they must demand empirically validated approaches that take into account not only the design of the curriculum, but the way the content is to be communicated and is to be implemented by teachers" (Dixon & Carnine, 1990, p. 1). Cawley, Baker-Kroczynski, and Urban (1992) have challenged educators to "modify the way we teach mathematics so it offers the child a different way to learn" (p. 41).

To meet that challenge, research-based instructional materials are needed that facilitate use of effective teaching procedures. To be effective for mainstream use, alternative materials must not only be appropriate for students with diverse learning needs (Dixon & Carnine, 1990) but must also be manageable within the grouping arrangements commonly used for instruction in regular classrooms (Gerleman, 1987; Good, Grouws, & Mason, 1990; Good, Grouws, Mason, Slavings, & Cramer, 1990).

### An Instructional Alternative in Mathematics

The *Strategic Math Series* is a scripted curriculum for teaching basic arithmetic facts to students with learning problems. Development of the series followed Mercer and Miller's (1992c) review of the literature and identification of critical components of effective mathematics instruction.

The series currently includes field-tested teaching manuals for addition (Miller & Mercer, 1991a), subtraction (Miller & Mercer, 1991b), multiplication (Mercer & Miller, 1992b), and division (Mercer & Miller, 1992a).

In the Strategic Math Series, instruction follows a concrete to representational to abstract (C-R-A) teaching sequence based on levels of human understanding described by both cognitive and behavioral learning theorists (Bruner, 1966; Gagné, 1977; Piaget, 1960; Reisman, 1982; Underhill, 1977; Underhill, Uprichard, & Heddens, 1980). At the concrete level, students use three-dimensional objects to solve problems accurately. Tallies replace concrete objects at the representational level. Between the representational and abstract levels, students learn a mnemonic strategy (D--Discover the sign; R--Read the problem; A--Answer, or draw and check; W--Write the answer) that links conceptual and procedural knowledge. At the abstract level, students use the DRAW strategy to figure out answers that are not immediately recalled.

The Strategic Math Series is designed to facilitate effective teaching through explicit, teacher-directed instruction. Teaching scripts combine the theories of curriculum design found in the Direct Instruction Model (Bereiter & Engelmann, 1966; Engelmann & Carnine, 1982;

Gersten, Carnine, & Woodward, 1987) with the cognitive approach validated for the Learning Strategies Curriculum (Deshler & Lenz, 1989; Deshler & Schumaker, 1986; Deshler, Schumaker, Alley, Warner, & Clark, 1982; Deshler, Schumaker, Lenz, & Ellis, 1984; Schumaker, Deshler, Alley, & Warner, 1983). Scripted procedures include (a) give an advance organizer, (b) describe and model the process, (c) conduct guided practice, (d) conduct independent practice, (e) conduct problem-solving practice, and (f) provide positive and corrective feedback.

Procedures for setting goals, teaching to mastery levels, providing feedback, and training students to generalize also reflect the multitheoretical foundation of the series. Challenging performance goals (Locke & Latham, 1990; Locke, Shaw, Saari, & Latham, 1981) are specified for mastery levels of automatic responding (Hasselbring et al., 1987; Kirby & Becker, 1988; Lovitt, 1989). Elaborated feedback is based on frequent monitoring of student performance (Christenson et al., 1989; Fuchs, 1986; Gersten et al., 1987; Good & Brophy, 1986; Kline, Schumaker, & Deshler, 1991; Slavin & Madden, 1989; Wang, 1987). Generalization to realistic problems is systematically programmed throughout instruction (Ellis, Lenz, & Sabornie, 1987a, 1987b; Stokes & Baer, 1977).

Twenty-two special education teachers, using the scripted manuals of the Strategic Math Series during field testing with 109 elementary students, required minimal training to implement the program in self-contained and resource classrooms. In both small- (fewer than 7) and large-group (7-18) arrangements, students with disabilities acquired computational skills; increased their rates of computation; generalized skills across settings, examiners, and tasks; generalized skills to word problems; and demonstrated independent use of the mnemonic strategy for solving difficult problems (Mercer & Miller, 1992c).

In terms of effective teaching, the design of the Strategic Math Series incorporates all of the best practices for mathematics instruction identified by Mercer and Miller (1992c). The advantages of the Strategic Math Series over instruction that follows a basal tradition are outlined in Table 3.

Effective and efficient for students with disabilities during field testing in special education and resource settings, the Strategic Math Series may also be appropriate for large-group instruction in regular classrooms that include mainstreamed students with disabilities. Through explicit, systematic, continuously monitored instruction, students with disabilities may be able to reach mastery

Table 3
Comparative Analysis of Two Mathematics Curricula

| Basal Curricula | INSTRUCTIONAL COMPONENTS | Strategic Math Series |
|---|---|---|
| - not scripted | EXPLICIT INSTRUCTION | + scripted |
| - rapid, cursory coverage | CONCEPTUAL UNDERSTANDING | + C-R-A sequence<br>+ cumulative review |
| - not scripted | FEEDBACK | + elaborated routine |
| - not systematic | GENERALIZATION | + programmed |
| - fluency omitted | TEACH TO MASTERY | + accuracy criterion<br>+ fluency criterion |
| √ nonspecific skill hierarchies | GOALS AND EXPECTANCIES | + student commitment<br>+ goal criteria |
| √ vague strategies<br>- limited abstract practice | TEACH PROBLEM SOLVING | + strategy instruction<br>+ guided practice<br>+ independent practice<br>+ problem-solving practice |
| √ not used systematically | MONITORING | + frequent, data based<br>+ charted |
| √ content/pace may be confusing or frustrating | POSITIVE ATTITUDE | + rationales<br>+ student involvement<br>+ mastery criteria<br>+ high interest<br>+ motivational games |
| √ pretest may not be provided<br>- spiraling content | APPROPRIATE CONTENT | + pretest included<br>+ mastery of basic facts |

*Note*: + incorporated √ questionable effect - missing levels in basic mathematical skills, maintain acceptable rates of learning in groups with normally achieving peers, and demonstrate independent performance during mainstream instruction.

Research was needed to investigate the effects of using the Strategic Math Series with heterogeneous groups of students in mainstream classrooms. When the Strategic Math Series is used by regular classroom teachers for large-group instruction, will students with disabilities become proficient at basic arithmetic facts? How will the performance of students with disabilities compare to the performance of normally achieving classmates receiving the same instruction?

## Statement of the Problem

The problem under investigation is the effectiveness of using Multiplication Facts 0 to 81 (Mercer & Miller, 1992b) from the Strategic Math Series to teach multiplication to mainstreamed students with disabilities in large-group arrangements with normally achieving peers. Regular classroom teachers were trained to use the procedures and materials of the curriculum. Experimental questions are related to the effects on students' understanding of multiplication, computational fluency, and performance levels within a large group when classroom teachers followed the procedures outlined in Multiplication Facts 0 to 81 (Mercer & Miller, 1992b) from the Strategic Math Series for large-group instruction:

1. What are the effects on accuracy (in terms of criterion responding) for students with disabilities?

2. What are the effects on fluency (in terms of criterion responding) for students with disabilities?

3. Will the performance of individual students with disabilities be similar to the performance of normally achieving peers in the same group on measures of accuracy and fluency?

Since students with disabilities are often placed in mainstream settings for mathematics instruction, the problem under investigation is important for several reasons. First, regardless of setting, explicit instruction in the concepts and skills of multiplication meets the need for students' acquisition of an important mathematical skill. Second, monitoring students' rates of performance provides information about the effects of instruction on computational fluency. Finally, comparison of individual and group performance levels demonstrates whether students with disabilities are able to maintain acceptable progress in large groups in mainstream settings when instructional conditions are controlled within a scripted curriculum.

## Rationale

It is well established that all students need mastery of the prerequisite skills that are essential for successful problem solving in mathematics. Problem-solving tools include computational proficiency in the basic facts of addition, subtraction, multiplication, and division. Mastery of facts is "a major stepping stone to success in mathematics for estimation and for both written and mental computation" (Thornton & Toohey, 1985, p. 44).

The traditional teaching practices and materials used for early instruction often present serious obstacles for students with learning problems who may require more than rote learning and abstract practice to become proficient with basic facts. Research-based alternatives are needed that prompt teachers to provide and monitor instruction in a manner that will improve students' chances for success. The Strategic Math Series is a scripted curriculum for basic facts that incorporates the critical cognitive and behavioral components of effective mathematics instruction (Mercer & Miller, 1992c). Explicit teaching procedures are merged with the principles of strategic learning within a concrete to representational to abstract (C-R-A) sequence based on hierarchical levels of understanding.

Concrete, representational, and abstract levels of instruction in mathematics were first compared for relative effectiveness (Armstrong, 1972; Fennema, 1972; Prigge, 1978; Scott & Neufeld, 1976; Smith, Szabo, & Trueblood, 1980). Overall, results were inconclusive. Evans and Carnine (1990) investigated the use of a two-level sequence (concrete and abstract) and alternated the order of presentation. Although both sequences were effective, the abstract-first sequence was more efficient while the concrete-first sequence resulted in greater early transfer. St. Martin (1975) compared a three-level sequence that included concrete instruction to a two-level sequence that omitted concrete experiences. The sequences were equally effective. Sigda (1983) made a similar comparison and found that students performed significantly better with the entire three-level sequence.

Peterson (1987/1988) demonstrated the effectiveness of the three-level (C-R-A) sequence to teach place value to low-achieving students. In Sealander's (1990/1991) doctoral dissertation and several graduate studies conducted at the University of Florida (Avino, 1990; Clifton, 1990; Essinger, 1988; Furtado, 1989; Gayler, 1987; Kelly, 1986; Matson, 1988; Natzke, 1989; Reid, 1988; Robertson, 1986; Wiers, 1990), the effectiveness of the sequence was confirmed for other mathematical skills.

Using direct instructional procedures for strategy instruction has produced positive student outcomes in mathematical computation (Carnine & Stein, 1981; Cook & Dossey, 1982; Kameenui et al., 1986; Lloyd, Saltzman, & Kauffman, 1981; Thornton, 1978, 1990). Maintenance and generalization occurred when students learned "meaningful problem representations as well as procedural steps for solution" (Goldman, 1989, p. 47).

The Strategic Math Series is a scripted curriculum that blends theoretical principles and empirical findings from behavioral and cognitive approaches to learning. Cutting across traditional theoretical boundaries, developers of the series addressed mathematical content as well as "the way the content is to be communicated and is to be implemented by teachers" (Dixon & Carnine, 1990, p. 1). The curriculum was field tested and found to be effective for students with disabilities under the controlled conditions of special education settings. This investigation examined the effects on mainstreamed students with disabilities when the curriculum was used by regular classroom teachers under typical classroom conditions at the elementary level.

## Definitions

The following section presents the definitions of important terms as they are used in this investigation.

*Abstract level of instruction*. At the abstract level, a student solves mathematical problems without using objects or drawings.

*Accuracy* (acquisition). Accuracy building is a stage of learning in which the primary objective is to reduce errors (Wolking & Stoutimore, 1989). In this investigation, accuracy refers to the percentage of correct responses on untimed independent practice activities specified for each lesson in the instructional sequence.

*Arithmetic*. Arithmetic deals with the "properties and relationships of real numbers and of computations with them involving chiefly addition, subtraction, multiplication, and division" (Gove, 1986, p. 118).

*Basic facts*. Basic addition facts include whole number combinations in which each addend is less than 10. Basic subtraction facts include combinations with one digit numbers for both the subtrahend and the difference. Basic multiplication facts include combinations with two single digit factors. Basic division facts include combinations with the form $c \div a = b$ in which a and b are single digit numbers and $a \neq 0$ (Silbert et al., 1990).

*Behavioral learning theory*. In a behavioral theory, attention is on "the study and manipulation of organism-environment interactions" (Tawney & Gast, 1984, p. 9).

Cognitive learning theory. In a cognitive theory, the focus is on internal thinking processes that assist a learner in constructing "meaning from experiences" (Mercer, 1983, p. 54).

Concrete level of instruction. At the concrete level, a student uses objects to solve mathematical problems.

Criterion. A criterion is a specified level of performance to be achieved before beginning the next task.

Direct Instruction. Direct Instruction is a highly structured approach to teaching that includes:

> 1. An explicit step-by-step strategy.
> 2. Development of mastery at each step.
> 3. Strategy (or process) corrections for student errors.
> 4. Gradual fading from teacher directed activities toward independent work.
> 5. Use of adequate, systematic practice with a range of examples.
> 6. Cumulative review of newly learned concepts.
>
> (Gersten et al., 1987, p. 49)

Emotionally Handicapped (EH). An emotional handicap is a condition that exists to a marked degree, interferes with the learning process, and results in "persistent and consistent maladaptive behavior" (Florida State Board of Education administrative rules, 1983-1992, p. 319).

Fluency. Fluency building is a stage of learning in which the primary objective is to build speed and perform a movement easily (Wolking & Stoutimore, 1989). In this investigation, fluency refers to the number of correct and incorrect written digits on a timed probe of basic multiplication facts.

*Mainstreaming*. Mainstreaming is the "temporal, instructional, and social integration" (Hardman, Drew, Egan, & Wolf, 1990, p. 520) of exceptional students with their regular education peers in school settings.

*Mastery*. In the *Strategic Math Series*, the mastery criterion for accuracy is 80% during the C-R-A sequence and 90% on the posttest. The mastery criterion for fluency is 30 correct digits per minute with 2 or fewer errors.

*Mathematics*. Mathematics is "a science that deals with the relationship and symbolism of numbers and magnitudes and that includes quantitative operations and the solution of quantitative problems" (Gove, 1986, p. 1393).

*Multiple baseline*. Multiple baseline is a type of single subject research design in which several baselines are recorded simultaneously and the intervention is introduced sequentially into one baseline at a time (Tawney & Gast, 1984).

*Probe*. A probe is used to sample a target behavior for a specified period of time. In this investigation, a probe of basic multiplication facts in a see-write format was used to monitor the number of correct and incorrect responses written during a 1-minute counting period.

*Problem solving*. Problem solving is a process performed to: "(1) generate acceptable responses that are not automatically generated, (2) clarify an ambiguity or a discrepancy, (3) determine an unknown, (4) provide alternatives in given situations, or (5) generate situations or instances that do not presently exist" (Cawley et al., 1988, p. 165).

*Rate*. Rate refers to the number of times a behavior occurs per unit of time. For this investigation, rate was measured in terms of frequency (movements per minute).

*Representational level of instruction*. At the representational level, a student uses drawings to solve mathematical problems.

*Single subject research*. Single subject research is an investigative strategy designed to measure and document changes in the behavior of an individual subject over time through continuous monitoring of the target behavior and visual analysis of graphically displayed data.

*Special education*. Special education is a subsystem of general education that provides "educational experiences to students who are considered different or exceptional" (Ysseldyke & Algozzine, 1984, p. 473).

Specific Learning Disability (SLD).

> Specific learning disability is defined as a disorder in one (1) or more of the basic psychological processes involved in understanding or in using spoken or written language. Disorders may be manifested in listening, thinking, reading, talking, writing, spelling, or arithmetic. Such disorders do not include learning problems which are due primarily to visual, hearing or motor handicaps, to mental retardation, to emotional disturbance, or to an environmental deprivation. (Florida State Board of Education administrative rules, 1983-1992, p. 321)

Strategy. A strategy enables a student "to learn, to solve problems, and to complete tasks independently" (Deshler & Schumaker, 1986, p. 583).

Tool skill. A tool skill is a prerequisite skill required for performance of a task. The tool skill for this investigation was the rate of writing digits 0-9 in a think-write format.

## Delimitations

This investigation has five delimitations. First, the investigation is restricted geographically to Union County in the State of Florida. Second, it includes only second grade students in a public school. Third, only mainstream classrooms that included students with disabilities (exceptional students according to State of Florida guidelines) participated in the investigation. Fourth, regular classroom teachers provided all instruction and feedback. Fifth, multiplication was the only mathematical operation taught during this investigation.

## Limitations

Generalization of results is limited to students in Union County in the State of Florida. Caution should be exercised in extending the findings to students in other grade levels, in different types of instructional settings, or with different or more severe learning problems than those described in this investigation. Since the investigation focused on one specific mathematical skill, the findings should not be generalized to other skills without replication of results.

## Summary

Educational placement decisions may make elementary classroom teachers responsible for teaching mathematics to students with a wide variety of learning difficulties. Mathematical learning in general education classrooms can be improved through implementation of teaching practices that have been associated with student success. Unfortunately, effective teaching is neither prompted nor facilitated by many of the traditional approaches and materials used for elementary mathematics instruction. An alternative curriculum, the Strategic Math Series, has been developed that incorporates the critical components of effective teaching within a validated instructional sequence based on theories of human learning.

Can learning opportunities in mathematics be improved for students with identified learning difficulties in mainstream settings through the use of structured curriculum materials that facilitate effective teaching? This question was investigated through classroom-based research that monitored the individual performance of mainstreamed students with disabilities during instruction in mathematics. Second grade teachers used a scripted curriculum, Multiplication Facts 0 to 81 (Mercer & Miller, 1992b), from the Strategic Math Series for whole-class instruction in multiplication. Classroom groups included students with disabilities and their normally achieving peers. This investigation has direct implications for elementary teachers who are responsible for teaching mathematics to mainstreamed students with disabilities.

A review of professional literature relevant to this investigation is presented in Chapter II. Methodological procedures for conducting the investigation are described in Chapter III. The results of the investigation are reported in Chapter IV and discussed in Chapter V.

# CHAPTER II
# REVIEW OF RELATED LITERATURE

The purpose of this chapter is to review and analyze the professional literature related to two procedures for teaching mathematics that are integral to the design of the curriculum used in this investigation. Specific procedures include (a) concrete, representational, or abstract instruction in a mathematical skill and (b) direct instruction in computational solution strategies.

The chapter is divided into four major sections. First, criteria for selection and inclusion of studies are described. Second, issues related to computational proficiency are discussed. Third, issues related to levels of instruction are considered and followed by a review of research related to concrete, representational, and abstract instruction in mathematics. Finally, the principles of direct and strategic instruction are described and followed by a review of investigations into direct instruction of computational strategies.

## Criteria for Selection of Relevant Literature

To be considered for the literature review, studies were analyzed according to the following criteria:

1. The purpose of the research was to investigate the effects of concrete, representational, or abstract instruction in a mathematical skill or the effects of direct instruction in a computational solution strategy.

2. Mathematics performance was the dependent measure in studies that examined concrete, representational, or abstract instruction.

3. A specific whole number operation was the dependent measure in studies that investigated direct instruction in strategies for computation of arithmetic facts.

4. Elementary or middle school students were the subjects for investigations related to concrete, representational, or abstract instruction. Elementary students were the subjects in studies that investigated direct instruction in strategies for computation of arithmetic facts.

5. Studies were data based and included complete descriptions of subjects, procedures, and results.

Studies were considered for inclusion if they met all criteria and were conducted between 1980 and 1991. In addition, important studies conducted prior to 1980, doctoral dissertations, and graduate studies conducted through the Department of Special Education at the

University of Florida were examined for relevant findings. Sources used to search for relevant literature included (a) Current Index to Journals in Education (CIJE); (b) Educational Resources Information Center (ERIC); (c) Library Technical Information Service (LTIS); (d) Library User Information Service (LUIS); (e) bibliographic references from related books, articles, documents, and reports; (f) Dissertation Abstracts International, and (g) graduate level studies conducted through the Department of Special Education at the University of Florida.

Twenty-seven studies were located that satisfied the requirements set forth in the inclusion criteria. The use of concrete, representational, or abstract instruction was the investigative focus in 21 studies. Direct instruction in computational strategies for arithmetic facts was examined in six studies.

## Computational Proficiency

Mastering the basic facts of addition, subtraction, multiplication, and division is a major objective for students in the elementary grades. Educators recognize that mastery of the facts is a prerequisite for success at higher levels of mathematics (Anghileri & Johnson, 1988; Goldman, 1989; Grossnickle, Reckzeh, Perry, & Ganoe, 1983; Hasselbring et al., 1987; Silbert et al., 1990; Thornton &

Toohey, 1985). Unfortunately, teaching students to produce answers accurately and automatically is not always an easy task (Baroody, 1985). Throughout the history of mathematics, "hundreds of ideas have been tried, yet many students find that the learning of the facts . . . is an exceptionally difficult, if not impossible, task" (Brulle & Brulle, 1982, p. 34).

Two very different theories have been generated about how a fact answer is produced, and different types of instruction result from each theory. In one theory, a retrieval procedure is activated to locate and recite a fact that has been stored in memory (Ashcraft, 1985). Instruction based on the retrieval theory uses repetitive drill to form automatic associations. The drill theory approach dominated elementary mathematics instruction in the United States during the first part of the twentieth century (Sovchik, 1989).

In another theory, mathematics is viewed as a system of meaningful processes. Instruction based on the meaning theory emphasizes the development of strategies for figuring out unknown number combinations. William Brownell is an authority often associated with the instruction for meaning that occurred in the United States between the 1920s and 1940s (Baroody, 1985).

Partly in response to the launching of the Soviet Union's Sputnik in 1957, Americans became concerned about the quality of science and mathematics instruction in the United States. For the following two decades, federally funded experimental programs, known as new math, were developed and introduced into classrooms throughout the country. Most of the new math programs were based on guided discovery of mathematical structures, patterns, and relationships (Sovchik, 1989). New math programs were designed to teach understanding of mathematical principles that would result in improved computational and problem-solving performance (Souviney, 1989). Although various new math programs had similar and precise objectives, classroom implementation was much less clear.

By 1975, the American public was discouraged with the results of the experimental programs. Citing poor performance levels throughout the country, critics of the programs called for a back to basics movement. Back to basics was interpreted by many as a return to the drill theory type of computation instruction that was prevalent during the early part of the century (Sovchik, 1989).

Mathematical performance has continued to deteriorate into the 1990s (Tindal & Marston, 1990). Again, the American public is concerned, and the National Council of

Teachers of Mathematics (NCTM, 1989) has issued a set of standards to guide major reform efforts. The consistent message throughout the standards is that students must understand mathematical processes in order to communicate in the language and ideas of mathematics.

Computational proficiency receives major emphasis in the NCTM standards for early instruction (NCTM, 1989). A problem-solving approach is recommended to develop conceptual understanding of mathematical operations and strategies for independent performance. Memorization and practice for speed are suggested only after students understand concepts and have developed efficient strategies for deriving unknown answers.

Moser (1988) suggested that a child employs an "idiosyncratic choice procedure that at one time uses a retrieval procedure and at other times the production system" (p. 133) for answering computation problems. If Moser is correct, then maximum benefit may accrue from instruction that emphasizes the meaning of whole number operations, mastery of efficient solution strategies for unknown facts, and practice for automatic responses.

The meaning of mathematical operations can be understood through experiences with manipulative and visual representations of the processes. Conceptually based strategies for solving abstract problems can be linked to the manipulative and visual representations. Accurate and automatic performance can be facilitated through fluency practice at the abstract level.

Descriptions of children's informal strategies (Baroody, 1985) provide valuable insight into how children understand and solve problems in addition (Baroody, 1987; Siegler, 1988), subtraction (Carpenter & Moser, 1984), and multiplication (Anghileri, 1989; Cooney, Swanson, & Ladd, 1988; Hope & Sherrill, 1987). Pressley (1986) defined a "good strategy user" (p. 140) as one who (a) knows about strategic procedures, (b) selects appropriate strategies, (c) executes strategies independently and efficiently, and (d) monitors strategy use.

Since students with learning problems rarely use solution strategies either effectively or efficiently on their own (Blankenship & Baumgartner, 1982), they need direct and systematic instruction to understand mathematical concepts and become good strategy users as defined by Pressley (1986). Learning theorists have identified several levels of understanding that can be used to guide instruction in the concepts that underlie procedures for performing mathematical operations.

## Concrete, Semiconcrete, and Abstract Instruction in Mathematics

Decisions about what and how to teach are influenced by beliefs about how children learn. Many theories have been proposed to describe the process of human learning. In some theories, the individual is viewed as a passive recipient of knowledge. In other theories, the learner must be actively involved in the process in order to gain knowledge. One recurring theme throughout many different theories is that learning progresses through hierarchical levels of understanding. Cognitive theorists such as Piaget (1960) and Bruner (1966) have described movement through each stage of learning. Gagné (1977) provided a behavioral perspective on children's learning.

Piaget (1960) described four levels of intellectual development that occur within a "dynamic, nonstatic evolution of newer and more complex mental structures" (Post, 1988, p. 7). According to Piaget, all children pass through sensorimotor, preoperational, concrete operational, and formal operational stages of intellectual development. The sensorimotor stage occurs from birth to approximately two years of age. During that stage, a child progresses from reflex actions and motor activities to the beginning of symbolic thought (Souviney, 1989). The child then enters a preoperational stage that lasts until about seven years of age. A preoperational child understands the purposes of symbolic thought but cannot mentally manipulate symbols. The child still uses egocentric thought, lacks logical analysis, and focuses on single attributes (Sovchik, 1989). By the concrete operational stage (about age 7), a child can use logical thought but only at a concrete level. Finally, at the formal operations stage (beginning at 11 or 12 years of age), a child is capable of logical, hypothetical, and abstract thought (Underhill, 1977). Children move through the stages at different rates depending on physiological, mental, and maturational variables.

Bruner (1966) described levels of understanding that closely parallel Piaget's (1960) stages of intellectual development. Bruner explained that ideas or concepts can be experienced, or known, at three different levels. At the enactive level, understanding occurs within motor or concrete experiences (Sovchik, 1989). At the iconic level, reality can be understood through visual representations (Post, 1988). Finally, language and abstract symbols can be understood at the symbolic level. All three levels are used sequentially for new learning (Underhill, 1977).

Gagné (1977) took a more behavioral view of the learning process and focused on what was learned rather than on how learning took place. According to Gagné, new learning is dependent upon mastery of prerequisite concepts and skills identified through detailed task analysis.

Underhill (1977) applied the cognitive and behavioral theories about learning to mathematics. Noting that most elementary mathematics instruction occurs with children in Piaget's (1960) preoperational and concrete operational stages, Underhill emphasized the importance of concrete and semiconcrete experiences during early instruction. He related Bruner's (1966) enactive, iconic, and symbolic levels of knowing to concrete, semiconcrete, and abstract instruction in mathematics. Prerequisite concepts and skills (Gagné, 1977) were interpreted as experiences at the concrete level that developed readiness for learning at the semiconcrete level and, finally, at the abstract level.

### The Concrete Level

At the concrete level, the movement of three-dimensional objects is used to solve problems. By relating visual representations and abstract symbols to the concrete activities, a child is able to experience concepts at all three of Bruner's (1966) levels of knowing--enactive, iconic, and symbolic (Underhill, 1977).

The use of concrete, manipulative materials to teach mathematics has a long history. Pestalozzi encouraged the use of concrete objects as early as the 19th century (Sowell, 1989). During the early 20th century, Montessori developed materials to give children experiences with real objects (Fuys & Tischler, 1979). Suggestions for using manipulative activities have been included in instructional materials since the 1930s. In the recent NCTM standards (NCTM, 1989), it is recommended that students should understand the numeration system and place value through concrete experiences prior to symbolic tasks.

Although the importance of concrete learning in mathematics has been documented (Mercer, 1992; Post, 1988; Sowell, 1989; Underhill et al., 1980), the concrete level of instruction rarely receives the same emphasis as the other stages. Most classroom instruction takes place at the semiconcrete and abstract levels following a few exploratory activities with concrete objects.

### The Semiconcrete (Representational) Level

Underhill's (1977) semiconcrete level, also known as representational, serves as a bridge between concrete experiences and the symbolism of the abstract level. At the representational level, objects are replaced by either pictures of the objects or tallies representing the objects. According to Underhill, representational experiences facilitate understanding at the iconic and symbolic levels.

### The Abstract Level

At the abstract level, numerical problems are solved using only symbols. Neither objects nor drawings are involved. Instruction at the abstract level includes rules, algorithms, and rote practice. Since only symbolic understanding is possible at the abstract level (Underhill, 1977), success is dependent upon a clear and unambiguous transition from manipulative experiences to symbolic representation.

## Studies Comparing the Concrete, Representational, and Abstract Levels in Mathematics Instruction

Although it is generally accepted that understanding mathematics at the concrete and representational levels is essential for successful performance at the abstract level, early studies emphasized the relative effectiveness of the various levels rather than a sequence of instruction. Five studies, conducted between 1972 and 1980, were located that compared the effectiveness of mathematics instruction at different levels (Table 4). The studies are discussed and analyzed in terms of subject characteristics, experimental procedures, research designs and measurement methods, maintenance and generalization measures, and results. Elements relevant to the present investigation are analyzed in detail.

Table 4

*Review of Studies Comparing the Concrete, Representational, and Abstract Levels in Mathematics Instruction*

| Author(s) | Subjects | Comparison | Skill |
|---|---|---|---|
| Armstrong (1972) | 1. 20 TMR (M.A.=2-4)<br>2. 67 EMR (M.A.=5.8-11.9) | manipulative vs. nonmanipulative | 1. numeral quantity association<br>2. conversion of quantity<br>3. numeral identification<br>4. counting |

Purpose: To investigate the effectiveness of manipulative and nonmanipulative modes of instruction at different levels of cognitive development and representational thought.

Results: Across cognitive levels, better learning occurred for mathematics that required representational thought (numeral quantity association) with the manipulative mode of instruction.

| | | | |
|---|---|---|---|
| Fennema (1972) | 95 second graders | concrete vs. symbolic | multiplication as union of equivalent disjoint sets |

Purpose: To investigate differences in learning using either a concrete or symbolic model of instruction.

Results: For direct recall, both models of instruction were effective. For transfer, students who used the symbolic model outperformed students who used the concrete model.

Table 4--Continued

<table>
<tr><th>Author(s)</th><th>Subjects</th><th>Comparison</th><th>Skill</th></tr>
<tr><td>Scott & Neufeld (1976)</td><td>145 second graders</td><td>manipulative vs. pictorial<br>----------------<br>abstract served as control for other 2 conditions</td><td>multiplication</td></tr>
<tr><td colspan="4">Purpose: To compare the contribution of concrete and pictorial materials to mathematical concept formation.</td></tr>
<tr><td colspan="4">Results: Manipulative objects and pictorial representations were equally effective for acquisition of concepts.</td></tr>
<tr><td>Prigge (1978)</td><td>146 third graders</td><td>nonmanipulative instruction vs. two-dimensional manipulative devices vs. three-dimensional objects</td><td>geometric concepts</td></tr>
<tr><td colspan="4">Purpose: To compare the effectiveness of two-dimensional and three-dimensional manipulative devices to each other and to a condition without manipulatives for teaching geometric concepts.</td></tr>
<tr><td colspan="4">Results: The three-dimensional manipulative condition was significantly more effective than the other two conditions for low-achieving students.</td></tr>
</table>

Table 4--Continued

| Author(s) | Subjects | Comparison | Skill |
|---|---|---|---|
| Smith, Szabo, & Trueblood (1980) | 66 first and second graders | manipulative vs. graphic vs. abstract | linear measurement |

Purpose: To compare manipulative instruction in linear measurement to graphic and abstract instruction in linear measurement.

Results: Manipulative instruction was significantly better than graphic instruction for acquisition of linear measurement skills. No significant difference was found between manipulative and abstract instruction.

Note: TMR: Trainable Mentally Retarded
EMR: Educable Mentally Retarded
M.A.: Mental Age

## Description of Studies

The investigative purpose in all of the studies summarized in Table 4 was to compare the effectiveness of instruction at various levels in the concrete to abstract instructional sequence. Armstrong (1972), Prigge (1978), Scott and Neufeld (1976), and Smith et al. (1980) used the term manipulative to refer to the level described by Fennema (1972) as concrete. The representational level was described as nonmanipulative by Armstrong (1972), graphic by Smith et al., (1980) and pictorial by Scott and Neufeld (1976). For the abstract level, Fennema (1972) used the term symbolic.

Armstrong (1972) conducted two studies to compare manipulative and nonmanipulative intervention conditions for teaching various mathematical concepts to 87 students labeled as Trainable (TMR) and Educable Mentally Retarded (EMR). Fennema (1972) assigned 95 second graders to either a concrete or symbolic treatment group for instruction in premultiplication concepts. Scott and Neufeld (1976) compared concrete and pictorial instruction in multiplication with 145 second graders. Prigge (1978) compared a traditional worksheet approach to using two-dimensional or three-dimensional manipulative objects for teaching geometric concepts to 146 third graders. Smith et al. (1980) assigned 66 first and second graders to a manipulative, graphic, or abstract treatment group for instruction in linear measurement.

## Subject Characteristics

In four of the five studies (Fennema, 1972; Prigge, 1978; Scott & Neufeld, 1976; Smith et al., 1980), subjects were elementary students without disabilities. Armstrong (1972) was the only investigator to include students with disabilities.

Armstrong (1972) described 20 students identified as Trainable Mentally Retarded (TMR) and 67 students identified as Educable Mentally Retarded (EMR). Students' mental ages ranged from 2 to 11.9 years. The TMR students in the first study were attending a summer session at a university laboratory school. All other subjects were students in public schools.

Sixty-six first and second graders served as subjects in the Smith et al. (1980) investigation. Fennema's (1972) 95 subjects and Scott and Neufeld's (1976) 145 subjects were second graders. Prigge's (1978) sample included 146 third graders.

## Experimental Procedures

All of the investigators used random assignment to place students, classes, or teachers in treatment conditions. Armstrong (1972) randomly assigned students to either a manipulative or nonmanipulative condition. Following screening for prerequisite skills, Fennema (1972) randomly assigned students to one of eight groups that then received either a concrete or a symbolic mode of instruction. Scott and Neufeld (1976) randomly assigned manipulative, pictorial, and abstract modes of instruction within three schools. Prigge (1978), following screening for ability blocking, randomly assigned subjects, teachers, and treatments (nonmanipulative, two-dimensional manipulatives, three-dimensional manipulatives) to three classes within two participating schools. Smith et al. (1980) screened students for length conservation and then randomly assigned subjects to manipulative, graphic, or abstract conditions.

Implementation. Systematic, controlled instruction was used in four studies. Armstrong (1972) controlled teacher presentation strategies, verbal input, content, and criterion testing across intervention conditions. Each teacher was trained to present both manipulative and nonmanipulative instruction and taught for the same amount of time in each condition. In Fennema's (1972) study, the same teacher instructed all groups for 14 sessions using the same worksheets, problems, and drill games. Scott and

Neufeld (1976) provided an orientation for principals and teachers and held instructional strategies and content constant for all groups of subjects. Prigge (1978) used prepared programmed units and held concepts, time, number of teachers and aides, and instructional materials constant across conditions. Smith et al. (1980) did not describe attempts to hold content or instructional procedures constant across conditions.

Materials. Different types of materials were used to differentiate concrete, representational, and abstract levels of instruction. For the manipulative condition, Armstrong's (1972) subjects used plastic animals. Pictures replaced the objects in the nonmanipulative condition.

Fennema (1972) provided Cuisenaire rods for concrete instruction. In the symbolic condition, subjects used only statements of relationships or written numerical symbols.

In the manipulative condition, Scott and Neufeld's (1976) subjects used concrete objects. Abstract symbols were used only after students demonstrated understanding of concepts. In the pictorial condition, pictures replaced the objects used in the manipulative condition. Subjects in the abstract group served as a comparative control for the other two conditions and used neither objects nor pictures.

In the nonmanipulative condition, Prigge's (1978) subjects used a traditional textbook-worksheet format with paper-and-pencil practice. Two-dimensional objects included geoboards, georulers, and folded paper. Three-dimensional materials were geometric solids such as cubes and tetrahedrons.

Smith et al. (1980) used teacher demonstrations with concrete materials such as paper clip chains and dowel rods during the manipulative condition. Pictures, filmstrips, and charts were used in the graphic condition. Abstract instruction was limited to verbal instructions and explanations of skills.

Duration of instruction. All investigators described duration of instruction. Armstrong (1972) presented 20 autoinstructional lessons. Scott and Neufeld (1976) also provided 20 controlled lessons designed to be completed during 35 minute class sessions. In the Fennema (1972) study, instruction occurred during 14 sessions. Prigge (1978) provided 10 days of instruction using prepared programmed units in 60 minute class sessions. Smith et al. (1980) conducted 30 minute sessions for 4 consecutive days.

Teacher training. Teacher training was mentioned in two studies (Armstrong, 1972; Scott & Neufeld, 1976). Armstrong (1972) reported that each teacher was trained to provide manipulative and nonmanipulative instruction and taught for the same amount of time in each condition. Scott and Neufeld (1976) described an orientation session with specific responsibilities for administrators and teachers.

### Research Designs and Measurement Methods

Group research designs with random assignment were used in all of the studies. Armstrong (1972) used multivariate analysis of covariance with I.Q. and pretest scores as covariates to analyze pretest and posttest results from two treatment conditions. Fennema (1972) analyzed group means on one recall and two transfer tests with one-way analysis of variance for fixed effects. Scott and Neufeld (1976) used analysis of covariance with I.Q. and pretest scores as covariates to compare two modes of instruction. Prigge (1978) tested the significance of differences among mean scores using analysis of variance in a 3X3 factorial design. Smith et al. (1980) analyzed posttest scores using analysis of variance for treatment effect and t-tests for differences between groups. Dependent measures included numeral quantity association, conversion of quantity, identification of numerals, and counting (Armstrong, 1972); multiplication (Fennema, 1972; Scott & Neufeld, 1976); geometric concepts (Prigge, 1978); and linear measurement (Smith et al., 1980).

Only one investigator (Prigge, 1978) used a standardized instrument for pretest measurement. Other pretest measures and all posttest measurement instruments were designed by the investigators. Fennema (1972) reported content validity and reliability coefficients of .91, .92, and .93 for the three instruments used in her study. Scott and Neufeld (1976) reported face validity and a reliability coefficient of .94 for their measurement instrument. Smith et al. (1980) reported content validity and reliability coefficients of .58, .78, and .45 for three criterion instruments. Armstrong (1972) and Prigge (1978) also used researcher-constructed instruments but provided no validity or reliability information.

### Maintenance and Generalization Measures

Maintenance was not considered in any of the studies. Only one study provided information about generalization. Fennema (1972) reported that, on transfer tests, students in the symbolic (abstract) mode of instruction performed higher than students who used the concrete mode.

### Results

Armstrong (1972) concluded that, across cognitive levels, manipulative instruction was better for mathematical learning that required representational thought. Fennema (1972) found that students were able to recall a mathematical idea following both concrete and symbolic instruction, but students in the symbolic group had better scores on transfer tests. Scott and Neufeld (1976) reported that pictorial and manipulative instruction were equally effective for acquisition of multiplication concepts. Prigge (1978) found that using three-dimensional manipulatives was significantly more effective than using two-dimensional manipulatives or omitting manipulatives when teaching geometric concepts to low-achieving students. Smith et al. (1980) concluded that manipulative instruction was superior to graphic instruction for acquisition of linear measurement skills. No differences were reported between manipulative and abstract modes of instruction.

Overall, the results are inconclusive. From the studies reviewed, it is not possible to make definitive statements about the relative effectiveness of concrete, representational, or abstract instruction in mathematics. Combining levels of instruction has been another research focus related to concrete, representational, and abstract learning in mathematics.

## Order of Presentation for Concrete and Abstract Instruction in Mathematics

In a recent study related to instructional levels in mathematics, Evans and Carnine (1990) used two levels (concrete and abstract) and compared their order of presentation during instruction in strategies for subtraction (Table 5). Procedures and results relevant to the use of a two-level instructional sequence are analyzed.

Table 5
Study Comparing the Order of Presentation of Concrete and Abstract Levels of Instruction in Mathematics

| Author | Subjects | Comparison | Skill |
|---|---|---|---|
| Evans & Carnine (1990) | 26 special education and Chapter 1 second and third graders | manipulative strategy followed by algorithm strategy<br>vs.<br>algorithm strategy followed by manipulative strategy | subtraction with borrowing |

Purpose: To compare the effectiveness and efficiency of two instructional sequences.

Results: Both sequences were effective. The concrete-first sequence was less efficient but produced greater early transfer than the abstract-first sequence.

Description of the Study

Evans and Carnine (1990) compared two sequences of instruction for developing conceptual and procedural knowledge of borrowing in subtraction. Concrete and symbolic (abstract) representations were used with special education and Chapter 1 elementary students. Students were randomly assigned to groups that received both types of instruction in reverse order. In the concrete to abstract group, students learned a manipulative strategy followed by an algorithm strategy for solving subtraction problems. The order was reversed for the abstract to concrete group. The experimental questions were concerned with both the effectiveness and the efficiency of instruction.

### Subject Characteristics

Subjects were 26 special education and Chapter 1 second and third graders in two schools. Subjects in each class were divided into special education and Chapter 1 groups that were further subdivided into male and female subgroups. Percentile differences between groups on a standardized mathematics instrument were analyzed with a one-way analysis of variance. Significant differences were not found, and groups were randomly assigned to two strategy conditions: manipulative instruction followed by instruction in algorithms or instruction in algorithms followed by manipulative instruction.

### Experimental Procedures

Students were screened on subtraction and place value. If students answered five or more of eight subtraction problems correctly, they were eliminated from final data analysis. Place value errors were remediated. Subtraction probes were administered daily during the study. A qualitative measure was used to evaluate students' level of understanding of the process. Students were also interviewed about their preferences for one sequence or the other. Ninety timed lessons were included in the study. Lessons for both strategies were adapted from commercial series. Interobserver reliability was reported for 24 lessons. The average discrepancy was reported as 8.25 seconds per lesson. Three graduate students, using scripted lessons, served as instructors and were rotated among groups. A checklist was used daily to monitor adherence to lesson scripts. A posttest was administered the day following completion of the intervention.

### Research Design and Measurement Methods

Means and standard deviations were reported for the posttests and maintenance checks on both concrete and abstract representations. Means, standard deviations, and range were reported for duration of instruction to criterion levels. A two-way repeated measure analysis of variance was conducted to test for interaction between treatment group and treatment phase for duration of instruction. Interview data were summarized for each group.

### Maintenance and Generalization Measures

A maintenance check, with the same items as the posttest, was administered 3 weeks after the study was concluded. At 3 weeks, there were no significant differences between groups. Transfer of knowledge was measured from the first to the second representation. Students in the concrete-first group transferred their understanding to algorithm instruction more easily than students in the abstract-first group transferred knowledge to concrete manipulations. Evans and Carnine (1990) noted that students in the concrete-first group received 58% more initial instruction than students in the abstract-first group. In addition, the concrete-first students were exposed to symbolic representations during initial instruction, but abstract-first students had no experience with concrete objects during initial instruction.

### Results

On both the posttest and maintenance check, there were no significant differences between groups. However, it was found that initial instruction with abstract representation was more efficient than learning first with a concrete representation. During the qualitative interview, 46% of the students in the abstract-first group correctly explained reasons for borrowing; only 23% of the students in the concrete-first group gave the same response. The incorrect practice of inverting the ones digit instead of borrowing was not observed for any abstract-first students following instruction, but 30% of the concrete-first students continued to invert. Students reported satisfaction with both methods of learning to borrow following presentation of the first strategy. Following the second strategy, 62% of the concrete-first group reported a preference for manipulative objects, whereas 85% of the abstract-first group preferred symbolic representations.

Evans and Carnine (1990) concluded that although conceptual and procedural learning occurred in both two-level sequences (concrete to abstract and abstract to concrete), instruction was more efficient when abstract instruction was presented first. However, average scores for both groups were fairly low on both the posttest (68%) and maintenance checks (60%). Perhaps the low scores for both groups indicate that students need explicit instruction to make the transition between concrete and abstract experiences. Since early transfer was greater for students who received concrete instruction first, instructional effectiveness might be improved by adding a transitional (representational) level to sequences that begin with concrete instruction.

## The Concrete to Representational to Abstract Instructional Sequence in Mathematics

Four doctoral dissertations were located that investigated the entire three-level sequence of concrete to representational to abstract (C-R-A) instruction (Table 6). The studies are discussed and analyzed in terms of subject characteristics, experimental procedures, research designs and measurement methods, maintenance and generalization measures, and results. Elements relevant to the present investigation are analyzed in detail.

### Description of Studies

St. Martin (1975) taught multiplication and division of fractions to 99 fifth graders using either a three-level concrete to abstract or a two-level semiconcrete to abstract sequence. Sigda (1983) compared three-level concrete to abstract instruction to instruction without concrete experiences for teaching multiplication to 52 third graders. Peterson (1987/1988) compared the three-level concrete to abstract sequence to traditional abstract instruction in place value for 24 students with learning disabilities. Sealander (1990/1991) investigated the effects of discontinuing the three-level concrete to abstract sequence when correct responses were higher than incorrect responses (crossover) in subtraction.

Table 6

Review of Doctoral Dissertations Investigating the Concrete to Abstract Instructional Sequence in Mathematics

| Author(s) | Subjects | Sequence | Skill |
|---|---|---|---|
| St. Martin (1975) | 99 fifth graders | concrete to abstract vs. semiconcrete to abstract | multiplication and division of fractions |
| Purpose: To compare treatment and Piagetian level effects in two-level and three-level sequences. | | | |
| Results: Concrete to abstract and semiconcrete to abstract instructional sequences were equally effective for acquisition and retention. | | | |
| Sigda (1983) | 52 third graders | concrete to abstract vs. representational to abstract | multiplication |
| Purpose: To compare the three-level concrete to abstract instructional sequence to the representational to abstract sequence. | | | |
| Results: Students performed significantly better when a concrete level was included in the instructional sequence. | | | |

Table 6--Continued

<table>
<tr><th>Author(s)</th><th>Subjects</th><th>Sequence</th><th>Skill</th></tr>
<tr><td>Peterson (1987/1988)</td><td>24 elementary and middle school LD</td><td>concrete to abstract vs. abstract</td><td>place value</td></tr>
<tr><td colspan="4">Purpose: To compare the three-level concrete to abstract instructional sequence to an abstract only condition.<br>Results: On acquisition and retention measures, students performed significantly better in the concrete to abstract condition.</td></tr>
<tr><td>Sealander (1990/1991)</td><td>8 first and second graders<br>----------<br>5 LD<br>3 EH</td><td>discontinuance of the concrete to abstract sequence at crossover</td><td>subtraction</td></tr>
<tr><td colspan="4">Purpose: To investigate the functional relationship between skill maintenance and generalization when the instructional sequence is discontinued at the occurrence of crossover.<br>Results: Students acquired, maintained, and generalized subtraction skills when instruction was discontinued at crossover.</td></tr>
</table>

*Note*: EH: Emotionally Handicapped
LD: Learning Disabled

### Subject Characteristics

Regular education elementary students were the subjects in two studies (St. Martin, 1975; Sigda, 1983). Effects for students with learning disabilities were investigated in Peterson's (1987/1988) study. Sealander (1990/1991) included students labeled as learning disabled and emotionally handicapped in her study. The subjects in Sealander's (1990/1991) study were first and second graders. Sigda's (1983) subjects were in third grade. St. Martin's (1975) subjects were fifth graders. Peterson (1987/1988) described her subjects as elementary and middle school students ranging in age from 8 to 13.

### Experimental Procedures

Random assignment and group designs were used to answer research questions by three investigators (Peterson, 1987/1988; St. Martin, 1975; Sigda, 1983). Peterson (1987/1988) also conducted a preliminary study using a single subject multiple baseline design across 3 learning disabled subjects. Sealander (1990/1991) used a single subject multiple baseline design across subjects to investigate acquisition and maintenance of skills.

Implementation. St. Martin (1975) attempted to control for teacher effects by teaching both treatment groups himself and using teachers of the experimental and control classrooms as teaching assistants. Peterson (1987/1988) and Sealander (1990/1991) used teaching scripts in a direct instruction format to achieve consistency of implementation. Lessons included (a) an advance organizer, (b) demonstration and modeling of the skill, (c) guided practice, and (d) independent practice. Peterson (1987/1988) also controlled instructional time periods, and each teacher in the study taught students from both treatment groups. In Sigda's (1983) study, experimental and control teachers taught their own classrooms.

Students were taught in groups in the studies by St. Martin (1975), Sigda (1983), and Peterson (1987/1988). In Sealander's (1990/1991) study, subjects were instructed individually.

Materials. St. Martin (1975) described use of paper fraction pieces, Cuisenaire rods, poker chips, and geoboards during concrete instruction. Manipulatives such as plastic counters and other common classroom objects were used by Sigda's (1983) subjects. Peterson (1987/1988) provided plastic cubes, place value sticks, and teacher-made place value strips for manipulation during concrete instruction. Sealander's (1990/1991) subjects used peanuts, cubes, and pennies during the concrete phase of instruction.

Duration of instruction. The treatment period was 27 days in the St. Martin (1975) study and 38 days for Sigda's (1983) subjects. Peterson's (1987/1988) subjects received a total of nine lessons--three lessons at each level of instruction (concrete, representational, and abstract). In Sealander's (1990/1991) study, subjects received as many of nine sequenced lessons as necessary until correct responses were higher than incorrect responses for 2 consecutive days. The length of instructional periods ranged from 15 minutes (Peterson, 1987/1988) to 55 minutes (St. Martin, 1975).

Teacher training. Sigda (1983) conducted an introductory inservice session for both teachers, a training session for the teacher of the experimental group, and weekly meetings to review instructional methods. Peterson (1987/1988) provided extensive training to teachers involved in her study. The concrete to abstract sequence and effective teaching procedures were explained, and teachers were trained to implement direct instruction procedures to mastery levels. Sealander (1990/1991) trained teachers to implement instructional procedures to mastery levels during three 1-hour workshops.

### Research Designs and Measurement Methods

St. Martin (1975) compared treatment effects, Piagetian level effects, and their interaction effect on performance through a 2X2 factorial design. A standardized instrument was used for the pretest. Investigator-designed tests were used for posttest and retention measures.

Sigda (1983) used a posttest-only control group design to test nine hypotheses. Six hypotheses were tested using a t-test analysis. For analysis, acquisition or retention of a multiplication skill was the dependent variable and treatment was the independent variable. Analysis of variance was performed to test three hypotheses for interaction effects of treatments, gender, and IQ levels. Teacher observation was conducted to establish procedural reliability. Experts were used to establish face and construct validity as well as reliability of the teacher observation instrument. Alternate forms of teacher-made tests were used to measure acquisition and retention of multiplication skills. A standardized academic aptitude test was used to identify three IQ subgroups. The experimental group received a sequence of instruction that included manipulative objects, pictorial representations, and symbols. The teacher of the control group used standard basal textbook presentation and no manipulatives.

Peterson (1987/1988) conducted a pilot study using a single subject multiple baseline design across 3 elementary subjects with learning disabilities. The purpose of the pilot study was to test the investigator-designed learning activities for each level of instruction. The activities were presented to students in a scripted direct instruction format. Analysis of the data allowed Peterson (1987/1988) to conclude that all 3 subjects acquired and retained place value skills. During baseline, all subjects made more error responses than correct responses. During the intervention, crossover occurred at various points. One subject reversed correct and error responses during the concrete level of instruction. The other 2 subjects demonstrated crossover during the representational level of instruction. Based on the results of the pilot study, Peterson (1987/1988) conducted the group study to compare the three-level teaching sequence to one-level abstract instruction.

In Peterson's (1987/1988) group study, students were screened for eligibility and randomly assigned to treatment and control groups. In the one-level group, subjects received nine abstract lessons. Neither manipulative objects nor pictures were used during instruction. Subjects in the three-level group also received nine lessons. Three lessons were presented at each level of instruction. Three alternate forms of an investigator-designed posttest were administered immediately after instruction ended, 1 week later, and again 3 weeks later.

Peterson (1987/1988) used a 2X3 mixed design with one between (treatment) and one within (performance over time) group factor. Significant differences among the levels of the experimental treatments were tested with a multivariate analysis of variance and follow-up tests.

Sealander (1990/1991) investigated a means of making the three-level (C-R-A) sequence more instructionally efficient. She investigated the effects of discontinuing direct instruction and using practice only at the point when a student's correct responses exceeded error responses for 2 consecutive days. Sealander (1990/1991) designed teaching scripts, a screening device, a pretest-posttest, daily probe sheets, and a word problem generalization test to monitor continuous progress in a single subject multiple design across subjects format. Acquisition and maintenance data were subjected to visual analysis. Celeration slopes and other measures of learning were also computed. Generalization was evaluated by percentage correct on untaught word problems. An interrater agreement score of 98.7% was reported for 157 reliability checks. Procedural reliability scores were above 90% for all three teachers.

### Maintenance and Generalization Measures

St. Martin (1975) and Sigda (1983) administered retention checks 4 weeks after instruction ended. In her pilot study, Peterson (1987/1988) conducted periodic maintenance checks for 2 weeks following instruction. A retention probe was given at 3 weeks. Posttreatment measures conducted in different settings indicated skill generalization. In the group study, Peterson (1987/1988) measured retention and skill generalization 1 week after instruction ended. In Sealander's (1990/1991) study, maintenance was checked daily for 5 consecutive days, then intermittently for 4 more weeks. Generalization was measured on the first day of the study and again at the beginning and end of the maintenance condition.

### Results

St. Martin (1975) concluded that concrete to abstract and semiconcrete to abstract instructional sequences were equally effective for acquisition and retention of fractions concepts by students at Piagetian Levels III and IV. The students in Sigda's (1983) study performed significantly better when a concrete level was added to the instructional sequence for multiplication. No significant interactions were found between teaching methods and IQ levels or gender. Peterson (1987/1988) found that, on acquisition and retention measures, students performed significantly better in the concrete to abstract condition during instruction in place value. Sealander (1990/1991) concluded that students acquired, maintained, and generalized subtraction skills when instruction occurred in a concrete to abstract sequence and was discontinued at crossover.

All of the investigators except St. Martin (1975) confirmed that concrete instruction was a critical component of an effective instructional sequence in mathematics. Sigda (1983) and Peterson (1987/1988) found superior effects for sequences that included concrete, representational, and abstract instruction. Sealander (1990/1991) found that skills acquired during the early stages of the instructional sequence were maintained and generalized when direct instruction was discontinued at the point of crossover.

With information about the effectiveness of the three-level sequence for place value (Peterson, 1987/1988), subtraction (Sealander, 1990/1991), and multiplication (Sigda, 1983), further research is justified to confirm the effects for other mathematical skills. Additional studies are also needed to analyze the effects of the instructional sequence for students with other categories of disability.

## Related Studies on the Concrete to Representational to Abstract Sequence in Mathematics

Several related studies have been conducted by graduate students in the Department of Special Education at the University of Florida to investigate the effectiveness of the three-level (C-R-A) sequence for acquisition, retention, and generalization of various mathematical skills and with different types of students (Table 7). Eleven studies are discussed and analyzed in terms of subject characteristics, experimental procedures, research designs and measurement methods, maintenance and generalization measures, and results. Elements relevant to the present investigation are analyzed in detail.

### Description of Studies

In all of the studies, the experimental purpose was to investigate the effects of the three-level concrete to abstract instructional sequence on various mathematical skills and with different types of students. The mathematical skills included (a) place value (Gayler, 1987); (b) money (Kelly, 1986); (c) addition (Gayler, 1987; Robertson, 1986); (d) subtraction (Essinger, 1988; Gayler, 1987; Natzke, 1989; Wiers, 1990); (e) multiplication (Clifton, 1990; Matson, 1988); (f) division (Furtado, 1989); (g) fractions (Reid, 1988); and (h) multiplication word problem solving (Avino, 1990). Regular education students were included as well as students described as at-risk, low-achieving, Chapter 1, educable mentally handicapped, emotionally handicapped. learning disabled, physically impaired, and severely emotionally handicapped.

Table 7

Review of Graduate Studies Investigating the Concrete to Representational to Abstract Sequence in Mathematics

| Author | Subjects | Intervention | Skill |
|---|---|---|---|
| Kelly (1986) | 3 elementary<br>----------<br>2 LD<br>1 EMH | scripted concrete to abstract instruction | money (dimes, nickels, and pennies to $.50) |
| Results: The concrete to abstract instructional sequence was effective for teaching money skills. | | | |
| Robertson (1986) | 3 elementary<br>----------<br>LD | scripted concrete to abstract instruction | addition facts 10-18 |
| Results: The concrete to abstract instructional sequence was effective for acquisition and retention of addition facts. | | | |
| Gayler (1987) | 6 elementary<br>----------<br>Chapter 1 | scripted concrete to abstract instruction | 1. place value for ones and tens<br>2. addition facts 10-18<br>3. subtraction facts 0-9 |
| Results: The concrete to abstract instructional sequence was effective for teaching place value, addition, and subtraction. | | | |

Table 7--Continued

<table>
<tr><th>Author</th><th>Subjects</th><th>Intervention</th><th>Skill</th></tr>
<tr><td>Essinger<br>(1988)</td><td>3<br>elementary<br>----------<br>2 LD<br>1 EH</td><td>scripted<br>concrete to<br>abstract<br>instruction</td><td>subtraction facts<br>10-18</td></tr>
<tr><td colspan="4">Results: The concrete to abstract instructional sequence was effective for acquisition, retention, and generalization of subtraction facts.</td></tr>
<tr><td>Matson<br>(1988)</td><td>3<br>elementary<br>---------<br>2 LD<br>1 regular education</td><td>scripted<br>concrete to<br>abstract<br>instruction</td><td>multiplication<br>facts 0-5</td></tr>
<tr><td colspan="4">Results: The concrete to abstract instructional sequence was effective for acquisition and retention of multiplication facts.</td></tr>
<tr><td>Reid<br>(1988)</td><td>3<br>elementary<br>---------<br>2 LD<br>1 low-achieving</td><td>scripted<br>concrete to<br>abstract<br>instruction</td><td>fractions</td></tr>
<tr><td colspan="4">Results: The concrete to abstract instructional sequence was effective for acquisition and retention of fractions.</td></tr>
</table>

Table 7--Continued

<table>
<tr><th>Author</th><th>Subjects</th><th>Intervention</th><th>Skill</th></tr>
<tr><td>Furtado<br>(1989)</td><td>6<br>elementary<br>3 LD<br>2 EH<br>1 EMH</td><td>scripted concrete to abstract instruction</td><td>division facts</td></tr>
<tr><td colspan="4">Results: Students who learned division through a concrete to abstract instructional sequence that was discontinued when correct responses were more than error responses (crossover) performed as well as students who received the full instructional sequence. Students without prior knowledge of multiplication required the full sequence but acquired, retained, and generalized division as well as subjects who had prior knowledge of multiplication.</td></tr>
<tr><td>Natzke<br>(1989)</td><td>3<br>elementary<br>regular education</td><td>scripted concrete to abstract instruction + mnemonic</td><td>subtraction facts 0-9</td></tr>
<tr><td colspan="4">Results: The concrete to abstract instructional sequence combined with a mnemonic device was effective for teaching subtraction facts.</td></tr>
<tr><td>Avino<br>(1990)</td><td>4<br>middle school<br>1 LD<br>3 EH</td><td>scripted concrete to abstract instruction</td><td>multiplication word problem solving</td></tr>
<tr><td colspan="4">Results: The concrete to abstract instructional sequence was effective for acquisition of skills for solving and creating word problems. The sequence was not effective for retention of skills for creating word problems.</td></tr>
</table>

Table 7--Continued

| Author | Subjects | Intervention | Skill |
|---|---|---|---|
| Clifton (1990) | 4 elementary<br>----------<br>SEH | scripted concrete to abstract instruction + mnemonic | multiplication |
| Results: The concrete to abstract instructional sequence combined with a mnemonic device was effective for acquisition, retention, and generalization of basic multiplication facts for students with severe emotional handicaps. | | | |
| Wiers (1990) | 3 elementary<br>----------<br>1 LD<br>1 PI<br>1 at-risk | scripted concrete to abstract instruction + mnemonic | subtraction facts 0-9 |
| Results: The concrete to abstract instructional sequence combined with a mnemonic cue was effective for acquisition, retention, and generalization of subtraction facts. | | | |

Note:
EMH: Educable Mentally Handicapped
EH: Emotionally Handicapped
LD: Learning Disabled
PI: Physically Impaired
SEH: Severely Emotionally Handicapped

## Subject Characteristics

Regular education students were among the subjects in the studies conducted by Matson (1988), Natzke (1989), and Reid (1988). Avino (1990), Essinger (1988), Furtado (1989), Kelly (1986), Matson (1988), Reid (1988), Robertson (1986), and Wiers (1990) investigated the sequence with students classified as learning disabled. Furtado (1989) and Kelly (1986) included subjects labeled as educable mentally handicapped. Avino (1990), Essinger (1988), and Furtado (1989) used the sequence with students identified as emotionally handicapped. Clifton (1990) taught students with severe emotional disabilities. One student with physical impairments was included by Wiers (1990). Gayler (1987) and Wiers (1990) worked with Chapter 1 and at-risk students. All of the students were in elementary and middle school grades.

## Experimental Procedures

In eight of the studies (Avino, 1990; Essinger, 1988; Furtado, 1989; Gayler, 1987; Kelly, 1986; Matson, 1988; Reid, 1988; Robertson, 1986), investigators followed a direct instruction format to achieve consistency of implementation. Lessons included (a) an advance organizer, (b) demonstration and modeling of the skill, (c) guided practice, and (d) independent practice. Noting that direct instruction could be enhanced by the addition of a mnemonic strategy, Natzke (1989), Clifton (1990), and Wiers (1990) added the mnemonic SOLVE to the abstract level. The mnemonic refers to procedural steps for solving problems independently: S--See the sign; O--Observe and answer; L--Look and draw; V--Verify the answer; and E--Enter the answer.

Students were taught individually or in small groups in all studies. Instruction was planned and presented by the investigators. Materials used for concrete instruction included checkers, cubes, poker chips, pipe cleaners, popsicle sticks, teddy bear counters, unifix cubes, and edibles. Pictures, tallies, and number lines were used at the representational level. Worksheets were used for abstract instruction. The studies typically involved 9 or 10 lessons. Lessons were presented for each level of instruction (concrete, representational, and abstract). To move from one lesson to the next, students were required to meet predetermined criteria.

### Research Designs and Measurement Methods

Ten of the investigators (Clifton, 1990; Essinger, 1988; Furtado, 1989; Gayler, 1987; Kelly, 1986; Matson, 1988; Natzke, 1989; Reid, 1988; Robertson, 1986; Wiers, 1990) used a single subject design to establish a functional relationship between the intervention and behavior change. Multiple baseline across subjects was the most frequently used format for data collection and analysis. Avino (1990) examined pretest and posttest data to determine changes in performance.

### Maintenance and Generalization Measures

Maintenance and retention checks were typically administered at 1 or 2 week intervals. Wiers (1990) used an interval of 6 weeks. Essinger (1988), Furtado (1989), and Wiers (1990) monitored generalization of skills to word problems. Clifton (1990) monitored generalization across teachers and settings.

### Results

All investigators reported positive results using the three-level (C-R-A) sequence to teach a variety of mathematical skills to regular and special education students. Kelly (1986) found the sequence to be effective for money skills with students labeled as learning disabled and educable mentally handicapped. Robertson (1986) reported acquisition and retention of addition facts by students with learning disabilities. Gayler's (1987) Chapter 1 students learned place value, addition, and subtraction skills. Essinger (1988) reported that students with learning and emotional disabilities acquired, retained, and generalized subtraction facts. Matson's (1988) subjects, described as regular education and learning disabled, acquired and retained multiplication facts. Reid's (1988) subjects, low-achieving and with learning disabilities, acquired and retained fraction skills. Furtado (1989) reported that subjects with learning, emotional, and mental disabilities acquired, retained, and generalized division skills even without prior knowledge of multiplication. Avino (1990) found that students identified as learning disabled and emotionally handicapped acquired skills for solving and creating word problems, but skills for creating word problems were not retained.

Three investigators reported positive results when a mnemonic strategy was added to the three-level (C-R-A) sequence. Natzke (1989) found the combination to be effective for teaching subtraction facts to regular education students. Natzke (1989) specifically noted the occurrence of crossover following introduction of the mnemonic strategy at the abstract level. Clifton (1990) used the three-level sequence plus mnemonic strategy to teach multiplication skills to students with severe emotional disabilities. His students acquired, retained, and generalized skills across teachers and settings. Wiers

(1990) reported that students labeled as learning disabled, physically impaired, and at-risk acquired and retained subtraction facts and generalized skills to word problems.

Graduate studies conducted at the University of Florida confirm the effectiveness of direct instruction in a three-level (C-R-A) sequence for teaching place value, addition, subtraction, multiplication, division, fractions, word problems, and money to regular and special education students in controlled settings. The addition of a mnemonic strategy (SOLVE) to the sequence was also found to be effective for subtraction and multiplication with individuals and small groups of students. More investigation is warranted to confirm the effects of providing direct instruction in computational strategies to larger and more heterogeneous groups of students in mainstream settings.

### Direct Instruction in Strategies for Computation

The Direct Instruction Model, developed by researchers at the University of Oregon (Bereiter & Engelmann, 1966; Carnine, Silbert, & Kameenui, 1990; Engelmann & Carnine, 1982), is a "complex way of looking at all aspects of instruction--from classroom organization and management to the quality of teacher-student interactions, the design of curriculum materials, and the nature of inservice teacher training" (Gersten et al., 1987, p. 48). A 10-year federally funded study, Project Follow Through, concluded that primary students who received direct instruction showed greater gains in mathematics, reading, spelling, and language than students who received other approaches based on child development theory, Piaget's stages of learning, the language experience model, discovery learning, and open education (Silbert et al., 1990).

Direct instruction materials include scripted teaching procedures that have been validated for basic skill acquisition through more than 20 years of research. Procedures include (a) demonstration and modeling of explicit strategies, (b) specified mastery criteria, (c) systematic feedback and correction procedures, (d) guided and independent practice with a wide range of examples, and (e) cumulative review of newly learned information (Gersten et al., 1987). Effective teaching researchers (e.g. Rosenshine & Stevens, 1986) have confirmed the benefits of explicit instruction based on the principles of the Direct Instruction Model.

Researchers at the University of Kansas Institute for Research in Learning Disabilities (Deshler & Lenz, 1989; Deshler et al., 1982; Deshler et al., 1984; Schumaker et al., 1983) combined cognitive techniques and direct instructional procedures to develop the Learning Strategies Curriculum for low-achieving adolescents. Learning strategies are defined as "techniques, principles, or rules that enable a student to learn, to solve problems, and to complete tasks independently" (Deshler & Schumaker, 1986, p. 583).

Within a scripted instructional sequence, strategy teachers communicate rationales based on pretest data, use advance and post organizers, describe and model use of each mnemonically cued strategy, sustain high levels of student involvement, monitor progress toward mastery, and conduct systematic reviews. Teachers use performance data to provide feedback during controlled and grade-appropriate practice sessions. Structured generalization activities allow students to demonstrate independent strategy use. Since 1979, strategies have been developed and field tested to help students acquire and retain content information and demonstrate written competence.

Researchers at the Universities of Oregon and Kansas have contributed valuable information to the knowledge base on effective instruction. The effectiveness of providing systematic instruction in explicit strategies has been reliably demonstrated.

## Studies Related to Direct Instruction in a Computational Strategy

Six studies were located in which strategies for the operations of addition, subtraction, multiplication, or division were explicitly taught to elementary students (Table 8). In order to be considered for this section, studies had to include a description of the strategy as well as an indication that instruction was explicit and conducted by teachers or experimenters. In five studies, instruction was presented to either heterogeneous groups of students or to groups of low-achieving students in regular classrooms. One study with individualized instruction for students with disabilities was included to examine the instructional efficiency of continuously monitored strategy instruction. The studies are discussed and analyzed in terms of subject characteristics, experimental procedures, research designs and measurement methods, maintenance and generalization measures, and results. Elements relevant to the present investigation are analyzed in detail.

### Description of Studies

Thornton (1978) conducted parallel investigations with 47 second and 43 fourth graders to compare a thinking strategies approach to a traditional textbook approach for addition and subtraction (second grade) and multiplication and division (fourth grade). Performance was evaluated on easy and difficult facts, and comparisons were made between high-achieving and low-achieving students in each group.

Table 8
Review of Studies Using Direct Instruction in a Strategy for Computation

<table>
<tr><th>Author(s)</th><th>Subjects</th><th>Facts</th><th>Strategy</th></tr>
<tr><td>Thornton<br>(1978)</td><td>47<br>second graders</td><td>addition<br>subtraction</td><td>thinking<br>strategies</td></tr>
<tr><td></td><td>43<br>fourth graders</td><td>multiplication<br>division</td><td></td></tr>
<tr><td colspan="4">Purpose: To compare performances of groups that received thinking strategy instruction and groups that received traditional instruction.</td></tr>
<tr><td colspan="4">Results: On basic and harder facts, students at both grade levels who learned strategies outperformed students who received traditional instruction. Both high and low achievers learned strategies that were explicitly taught.</td></tr>
<tr><td>Carnine &<br>Stein II<br>(1981)</td><td>20<br>low-achieving<br>first graders</td><td>addition</td><td>counting<br>relationship<br>between facts</td></tr>
<tr><td colspan="4">Purpose: To compare the effectiveness of a strategy based on the counting relationship between successive facts to memorization of fact families.</td></tr>
<tr><td colspan="4">Results: Compared to memorization, students showed more learning and better retention with strategy teaching.</td></tr>
</table>

Table 8--Continued

| Author(s) | Subjects | Facts | Strategy |
|---|---|---|---|
| Lloyd, Saltzman, & Kauffman I (1981) | 4 LD 8-9 years of age | multiplication | preskill and attack strategy |
| II | 3 LD (same as I) | division | attack strategy |

Purpose: I To investigate whether training in preskills alone would result in acquisition of multiplication.

Results: I Preskills and strategy training were both necessary for acquisition of the skill.

Purpose: II To investigate whether teaching similar strategies for related skills would enhance or inhibit performance in both skills.

Results: II Acquisition of a second skill can be enhanced by teaching a similar strategy for a related class of items. Confusion between similar strategies did not occur.

| Author(s) | Subjects | Facts | Strategy |
|---|---|---|---|
| Cook & Dossey (1982) | 219 third graders | multiplication | thinking strategies |

Purpose: To compare the thinking strategies approach with the basal factor size approach.

Results: Compared to an approach based on size of factors, students learned facts more quickly with a thinking strategies approach.

Table 8--Continued

| Author(s) | Subjects | Facts | Strategy |
|---|---|---|---|
| Kameenui, Carnine, Darch, & Stein - I (1986) | 23 low-achieving first graders | subtraction | semiconcrete, line-drawing |

Purpose: To compare direct instruction strategy training to basal strategy training.

Results: Students who received direct instruction in the subtraction strategy outperformed students in the comparison group.

| Author(s) | Subjects | Facts | Strategy |
|---|---|---|---|
| Thornton (1990) | parallel studies<br>I - 70 first graders<br>II - 68 first graders | subtraction | various counting and noncounting strategies |

Purpose: To compare strategy training to drill instruction.

Results: Compared to a control group that received drill instruction, students who learned strategies knew more memorized facts and derived more answers to unknown facts.

Carnine and Stein (1981) compared a strategy based on counting relationships to memorization of basic addition facts with 20 first grade students. Lloyd et al. (1981) conducted two studies. In the first study, the investigators monitored the performance of 4 subjects with learning disabilities during preskill training for multiplication. Finding that preskills alone were not sufficient, they implemented strategy training for multiplication. In the second study, 3 students from the first study learned a similar strategy for division. Cook and Dossey (1982) compared a thinking strategies approach to a basal approach based on the size of factors for multiplication with 219 third grade students. Kameenui et al. (1986) compared direct instruction strategy training to basal strategy training for subtraction with 23 first graders. Thornton (1990) conducted parallel studies with 70 and 68 first graders to compare instruction in strategies to drill for subtraction facts.

### Subject Characteristics

In three studies (Cook & Dossey, 1982; Thornton, 1978, 1990), subjects were heterogeneous groups of students from regular classes. In two studies (Carnine & Stein, 1981; Kameenui et al., 1986), subjects were identified as low-achieving students. The subjects in the Lloyd et al. (1981) study were identified as learning disabled and attended a self-contained classroom. In three studies (Carnine & Stein, 1981; Kameenui et al., 1986; Thornton, 1990), the subjects were first graders. Thornton (1978) examined the performance of second and fourth graders. Cook and Dossey's (1982) subjects were in third grade.

Lloyd and colleagues (1981) described their subjects as children eight to nine years old.

### Experimental Procedures

Cook and Dossey (1982) randomly selected two third grade classes in each of six randomly selected schools. Three schools were then selected for each treatment group. Students assigned to the two groups were not statistically different. In two studies (Carnine & Stein, 1981; Kameenui et al., 1986), students were randomly assigned to treatment groups. Thornton (1978) pooled intact groups from multilevel classes. Thornton (1990) used matched groups of first grade students in two schools. Lloyd et al. (1981) selected subjects based on pretest assessment and used single subject procedures to monitor individual performance.

Implementation. Students were instructed in groups in five studies (Carnine & Stein, 1981; Cook & Dossey, 1982; Kameenui et al., 1986; Thornton, 1978, 1990). Lloyd et al. (1981) trained subjects individually. Instructional procedures were described in varying amounts of detail. Thornton (1978) was somewhat vague and referred the reader to the published materials used as teaching guides. However, explicit instruction was referred to in both the procedures and discussion. In Cook and Dossey's (1982) study, teachers were trained to use the procedures in the materials referred to by Thornton (1978). Carnine and Stein (1981) described teacher modeling, fading of prompts, feedback, and systematic correction procedures. Lloyd et al. (1981) provided teacher modeling, guided practice with verbalization, independent practice, and feedback through reinforcement.

Kameenui et al. (1986) described their direct instruction procedure as "clearly articulated teaching sequences that contain explicit, step-by-step teacher modeling and a means of assessing student mastery at each step of development" (p. 635). Systematic correction procedures were also described.

In Thornton's (1990) parallel studies, teachers used direct modeling, recording, and independent practice after mastery of concepts. Students in the strategies group also received cumulative review.

Various strategies were used in the investigations. Carnine and Stein (1981) used a strategy based on counting relationships. For example, if a student knew that $6 + 1 = 7$, the counting relationship could be used to figure out that $6 + 2 = 8$ and $6 + 3 = 9$.

Thornton (1978) described thinking strategies such as doubles + 1, sharing numbers, counting on, using tens, using time or money models, and rhymes. Cook and Dossey (1982) made several methodological changes in Thornton's (1978) design and attempted to replicate the effects. Changes included equating drill and practice time for both groups, lengthening the time between instruction and retention testing, and using younger students to avoid relearning effects.

Thornton's (1990) solution strategy approach was based on the idea that "when concept understanding for subtraction has been demonstrated and prior to drill for speed, teachers should intervene and actively teach children solution strategies for working out answers to unknown subtraction facts" (p. 247). Several strategies were described: show all using objects, mental counting up or back, mental imagery associations, related facts, 10-frames, and memorized facts.

Lloyd et al. (1981) trained students to point to a number they could count by, make marks for the other number, count by the count-by number one time for each mark, and write the answer. The strategy was referred to as an attack strategy because, with mastery of the strategy, a student could attack unknown problems.

Kameenui et al. (1986) taught students to read a problem, draw lines for the subtrahend, cross out the number subtracted, and count the remaining lines. Problems included numbers up to 19.

Materials. In two studies (Kameenui et al., 1986; Thornton, 1978), investigator-designed strategy materials were used by students in the experimental groups; basal materials were used by the comparison groups. Cook and Dossey (1982) compared published materials for instruction in thinking strategies to basal instruction. Carnine and Stein (1981) used verbal presentation. Lloyd et al. (1981) used verbal presentation, chalkboard practice, and worksheet exercises. In Thornton's (1990) parallel studies, subjects in both groups received the same worksheets during conceptual training and drill. Between those two stages, subjects in the experimental group received strategies training using investigator-designed worksheets.

Duration of instruction. There was great variation in the duration of instruction. Instruction ranged from a single instructional session lasting 110 minutes (Lloyd et al., 1981) to instruction that occurred throughout the school year (Thornton, 1990). In between the two extremes, instruction ranged from 1 to 9 weeks. Carnine and Stein (1981) provided 10 minutes of instruction daily for 7 days. In the study by Kameenui et al. (1986), subjects received 25 minutes of instruction daily for 6 days and 10-12 minutes on the seventh day. Thornton's (1978) subjects received 20 minutes of instruction for 3 consecutive days each week for 8 weeks. Cook and Dossey (1982) provided 20 minutes of daily instruction for 9 weeks.

Teacher training. In three studies (Cook & Dossey, 1982; Thornton, 1978, 1990), regular classroom teachers were trained to provide all instruction. All three investigators reported attempts to control for teacher and presentation effects through training or materials. Experimenters served as instructors in the Carnine and Stein (1981), Kameenui et al. (1986), and Lloyd et al. (1981) studies.

### Research Designs and Measurement Methods

In five investigations (Carnine & Stein, 1981; Cook & Dossey, 1982; Kameenui et al., 1986; Thornton, 1978, 1990) group designs and data analysis procedures were used to investigate the effects of strategy instruction. Carnine and Stein (1981) examined posttest differences on investigator-designed instruments. An accompanying study with preschoolers described the use of a Mann-Whitney procedure to test for significant differences. Interrater and procedural reliability data were not provided.

Cook and Dossey (1982) administered a pretest, two tests during instruction, a posttest, and a retention test.

Difference scores among tests were analyzed through one-way analysis of variance. Students in each treatment group were also divided into high and low aptitude subgroups based on a standardized test score. A two-way analysis of variance was used to examine differences. Procedural reliability was accomplished through observations and audio recordings. Based on observations, two classes were dropped from the study due to violation of experimental procedures.

Kameenui et al. (1986) analyzed training and maintenance scores. Analysis of variance was used to examine differences among treatment groups for each test.

Since random assignment was not possible, Thornton (1978) used groups as the unit of analysis at each grade level. Analysis of variance with repeated measures on the second factor (pretest, posttest, retention test) was performed for each operation with follow-up tests for differences within groups.

Thornton (1990) analyzed data from multiple interviews and written test data. Interview data were summarized by percentage of strategies used for correct and quick responses to various facts. Means and standard deviations were presented for written tests.

Lloyd et al. (1981) used a single subject multiple baseline across subjects design in a 110 minute instructional session. Probes were administered approximately every 5-9 minutes. The shortest training session for any student was 14 minutes, and the longest training session was 18 minutes. Interrater reliability measures were obtained on all 216 probe responses but were not reported.

### Maintenance and Generalization Measures

Students in the Kameenui et al. (1986) study were given maintenance tests at 1 day, 4 days, and 1 month after instruction. Lloyd et al. (1981) reported follow-up measures 4 days after training. Generalization was monitored throughout training. In Carnine and Stein's (1981) study, a delayed posttest was administered 4 days after the study ended. Thornton (1978) measured retention after 2 weeks and transfer at 3 and 7 month intervals. Thornton (1990) conducted a retention test 3 months after training. Cook and Dossey (1982) administered a posttest 8 weeks after instruction and a retention test almost 2 months after the posttest.

### Results

In the three studies that compared explicit strategy instruction to a basal approach (Cook & Dossey, 1982;

Kameenui et al., 1986; Thornton, 1978), students who received strategy instruction outperformed students who received traditional basal instruction. Cook and Dossey (1982) reported statistically significant differences in favor of the thinking strategies approach for three measurement intervals: pretest to test at 6 weeks, pretest to posttest, and pretest to retention test. Only pretest to the first test at 3 weeks was not significant. In addition, subgrouping by aptitude indicated more learning from the first test to the second test for high and low aptitude students in the strategy group than for students in the basal factor size group. Kameenui et al. (1986) found significant differences favoring the direct instruction strategy group on three measures: correct responses on numerical training problems, correct statements for pictorial problems, and skill maintenance. The final maintenance scores indicated some deterioration over time. Thornton (1978) found that, at both second and fourth grades, students who learned strategies correctly solved more problems (easy and difficult) than students in the traditional instructional group.

Thornton (1990) kept instruction the same for both groups except for strategy training in the experimental group. Both groups received the same instruction for conceptual understanding and drill. Thornton (1990) reported data from five interviews related to solution strategies and written tests. From the interview data, she concluded that strategy students used a greater variety of strategies to solve subtraction problems. On the written posttests, significant differences favored the strategy students on all types of problems. The trend was repeated on the retention measure. Thornton (1990) reported that investigators noted more willingness to attempt unknown problems among strategy students than among students in the control group.

Carnine and Stein (1981) found that, with training time held constant, students in the strategy group had learned 21 facts compared to 15 facts for students in the memorization group. Performance differences on the delayed posttest were significantly different and favored the strategy group. The investigators noted that, initially, strategy instruction was time-consuming but seemed to result in higher final accuracy levels.

Lloyd et al. (1981) investigated two related questions. Their first finding was that both preskill training and explicit strategy training are necessary for acquisition and generalization of multiplication skills. Results of the second experiment showed that students could rapidly acquire a related skill without confusion through instruction in a strategy similar to one they had previously learned.

## Summary

With trends toward mainstream placement of students with a variety of learning problems, interventions are needed that improve instructional conditions for hard-to-teach students and their teachers. Educational researchers have identified teaching behaviors consistently associated with student success. Guidance for implementing the behaviors in specific curriculum areas can be found in the professional literature. The findings of studies reviewed in this chapter confirm the advantageous results of implementing more effective teaching in mathematics at the elementary level.

The effectiveness of a three-level (concrete to representational to abstract) teaching sequence for mastery of mathematical concepts was demonstrated in 14 studies. The sequence was appropriate for acquisition and retention of money concepts, place value skills, addition, subtraction, multiplication, division, fractions, and word problems. Generalization was reported for subtraction, multiplication, and division. Subjects were described as regular education students; students with learning disabilities, emotional, mental, and physical handicaps; and students labeled as low-achieving, at-risk, and Chapter 1. In 12 of the 14 studies, individual performance was continuously monitored through single subject methodology. Even with extended baselines, performance generally did not improve until the C-R-A sequence was initiated. Across all single subject studies, the median percentage of overlap between baseline and intervention was 29% for correct responses and 20% for incorrect responses. Clearly, the three-level sequence is effective for acquisition of many mathematical skills and facilitates retention and generalization of new skills.

The benefits of explicit instruction in solution strategies for addition, subtraction, multiplication, and division were demonstrated in six studies with elementary students. Subjects included regular education students and students identified as learning disabled and low-achieving. In three of the six studies, instruction was presented to heterogeneous groups of students by regular classroom teachers. Individual performance measures were not reported in the five studies that utilized group designs, so it is impossible to determine relative rates of learning for low-achieving students. In the study that used a single subject design, individual data were reported for students with disabilities but not with respect to the performance of normally achieving students under the same conditions. However, in all cases, groups that received strategy training in computation outperformed groups that received traditional basal instruction or drill and memorization training.

From the studies reviewed in this chapter, it is clear that development of conceptual understanding for whole number operations is possible through a three-level explicit teaching sequence that includes concrete experiences, visual representations of the concrete experiences, and practice with abstract symbols. It is also established that direct instruction in computational solution strategies is more effective for independent performance than either traditional basal instruction or memorization and drill.

The Strategic Math Series is a recently developed and field tested curriculum that incorporates the principles of direct instruction and strategic learning within the three-level concrete to representational to abstract instructional sequence. In the studies reviewed in this chapter, the primary components of the Strategic Math Series were shown to be associated with improved learning outcomes in mathematics. The curriculum has been successfully field tested with hard-to-teach students and specialist teachers in restrictive settings. Since, in actual practice, regular classroom teachers are often responsible for teaching mathematics to students with learning problems, further research is warranted to investigate the effectiveness of the _Strategic Math Series_ for general classroom use with large heterogeneous groups of students.

# CHAPTER III
# METHOD

The purpose of Chapter III is to present the research methodology used to investigate three questions about large-group instruction in multiplication for students with disabilities (exceptional students) in mainstream settings. When classroom teachers follow the instructional procedures outlined in Multiplication Facts 0 to 81 (Mercer & Miller, 1992b) from the Strategic Math Series for large-group instruction:

1. What are the effects on accuracy (in terms of criterion responding) for students with disabilities?

2. What are the effects on fluency (in terms of criterion responding) for students with disabilities?

3. Will the performance of individual students with disabilities be similar to the performance of normally achieving peers in the same group on measures of accuracy and fluency?

Applied behavior analysis procedures (Cooper, Heron, & Heward, 1987) were used to collect, display, and analyze performance measures of accuracy and fluency during direct instruction in multiplication. Methodological issues are discussed in terms of (a) subject selection, (b) subject description, (c) description of teachers and teacher training, (d) setting, (e) instructional materials, (f) experimental procedures, (g) measurement instrumentation, (h) experimental design, (i) validity and reliability procedures, and (j) data analysis.

## Subject Selection

Subjects for this investigation were 155 second grade students in six mainstream classrooms. All six classrooms were scheduled to begin multiplication at the time of this investigation. Multiplication Facts 0 to 81 (Mercer & Miller, 1992b) from the Strategic Math Series replaced the basal unit on multiplication. The size of instructional groups ranged from 25 to 27 students.

Several inclusion criteria were in effect for this investigation. Parental or guardian permission was required for reporting results. Permission was obtained for 125 (81%) students. The other 30 students participated fully in all instruction, but their scores are not included in any measures reported for the investigation.

Since timed performance was monitored throughout the investigation, students were screened for the tool skill of writing digits (0-9) from memory under timed conditions. Screening also included two prerequisite skills for computing multiplication facts up to 81: counting consecutively to 81 and solving basic addition facts 0-18. Teachers administered the screening instrument according to scripted directions. The screening instrument and directions for administration are provided in Appendix A. Of the 125 students for whom permission was obtained, 11 students who wrote fewer than 30 digits per minute or scored below 70% on counting or solving basic addition problems were excluded from data analysis. Excluded subjects participated in all instruction, but their scores are not included in any measures reported for answering the experimental questions.

Among the students with permission slips and satisfactory levels of tool and prerequisite skills were 15 students with disabilities (exceptional students according to state of Florida guidelines). Two exceptional students were excluded after they demonstrated median error rates of zero during the baseline phase of monitoring. The excluded exceptional students participated in all instruction, but their performance is not reported since the experimental questions relate to exceptional students in the beginning stages (accuracy building) of learning multiplication.

After all exclusions, the final sample was comprised of 13 exceptional subjects labeled as having either a specific learning disability (SLD) or an emotional handicap (EH) and 99 normally achieving peers who received the same instruction. All subjects demonstrated proficiency on tool and prerequisite skills by writing 30 or more digits (0-9) per minute, counting to 81 with at least 70% accuracy, and solving addition problems with at least 70% accuracy. Screening results are summarized in Appendix B. Within each classroom, exceptional subjects are identified with a classroom number followed by a subject number.

### Subject Description

Smith et al. (1984) recommend guidelines for describing subjects with learning disabilities in published reports. In this investigation, those recommendations guide the individual descriptions of subjects with disabilities as well as the summary descriptions of their peers without disabilities in each classroom (Tables 9 through 14).

Exceptional subjects are described in terms of gender, age, ethnicity, socioeconomic status (SES), special education status (type of exceptionality), intelligence quotient (IQ), and achievement levels in mathematics. Except for IQ scores, similar information is provided in summary form for peers in each classroom.

For exceptional subjects, intelligence quotient scores were obtained from the Mental Composite Score of the Kaufman

Table 9
Description of Subjects--Classroom 1

| Total in Group | Permission Slips | Number Excluded | Number Described |
|---|---|---|---|
| 25 | 23 | 4 (screening) | 19 |

| Subject | Gender | Age | Ethnicity | Lunch Program | Status | IQ | Mathematics Achievement (CTBS-1992) |
|---|---|---|---|---|---|---|---|
| 1.1 | Female | 7.11 | Caucasian | Free | SLD | 125 K-ABC | 13 |
| 1.2 | Male | 8.6 | Caucasian | Free | SLD | 96 K-ABC | 9 |
| 1.3 | Male | 8.3 | Caucasian | Pay | SLD | 91 K-ABC | Not Available |
| Classroom 1<br>Other Group Members<br>N=16 | Female<br>8<br>Male<br>8<br>Total<br>16 | Median<br>8.1<br>Range<br>7.8-8.7 | African-American<br>4<br>Caucasian<br>12 | Free/Reduced Lunch<br>8<br>Pay<br>8 | Ch. 1<br>4 | ▨ | N=16<br>Median<br>69<br>Range<br>18-98 |

Note: CTBS: Comprehensive Test of Basic Skills (National Percentile)
SLD: Specific Learning Disability
K-ABC: Kaufman Assessment Battery for Children

Table 10

Description of Subjects--Classroom 2

| Total in Group | Permission Slips | Number Excluded | Number Described |
|---|---|---|---|
| 27 | 19 | 0 | 19 |

| Subject | Gender | Age | Ethnicity | Lunch Program | Status | IQ | Mathematics Achievement (CTBS-1992) |
|---|---|---|---|---|---|---|---|
| 2.1 | Male | 8.3 | Caucasian | Pay | SLD | 102 K-ABC | 76 |
| 2.2 | Male | 9.4 | Caucasian | Free | SLD | 72 K-ABC | 17 |
| Classroom 2<br>Other Group Members<br>N=17 | Female<br>9<br>Male<br>8<br>Total<br>17 | Median<br>8.4<br>Range<br>7.8-8.8 | African-American<br>1<br>Caucasian<br>16 | Free/ Reduced Lunch<br>5<br>Pay<br>12 | Ch. 1<br>1 | ▨ | N=17<br>Median<br>88<br>Range<br>25-99 |

Note: CTBS: Comprehensive Test of Basic Skills (National Percentile)
SLD: Specific Learning Disability
K-ABC: Kaufman Assessment Battery for Children

Table 11

Description of Subjects--Classroom 3

| Total in Group | Permission Slips | Number Excluded | Number Described |
|---|---|---|---|
| 26 | 25 | 4 (screening)<br>1 (baseline) | 20 |

| Subject | Gender | Age | Ethnicity | Lunch Program | Status | IQ | Mathematics Achievement (CTBS-1992) |
|---|---|---|---|---|---|---|---|
| 3.1 | Female | 8.8 | Caucasian | Pay | EH | 120 K-ABC | 90 |
| Classroom 3 Other Group Members N=19 | Female<br>10<br>Male<br>9<br>Total<br>19 | Median<br>8.5<br>Range<br>7.10-8.7 | African-American<br>3<br>Caucasian<br>15<br>Hispanic<br>1 | Free/ Reduced Lunch<br>11<br>Pay<br>8 | Ch. 1<br>2<br>Migrant Program<br>1 | | N=18<br>Median<br>69.5<br>Range<br>30-99 |

Note:
CTBS: Comprehensive Test of Basic Skills (National Percentile)
EH: Emotionally Handicapped
K-ABC: Kaufman Assessment Battery for Children

Table 12

Description of Subjects--Classroom 4

| Total in Group | Permission Slips | Number Excluded | Number Described |
|---|---|---|---|
| 26 | 22 | 2 (screening)<br>1 (baseline) | 19 |

| Subject | Gender | Age | Ethnicity | Lunch Program | Status | IQ | Mathematics Achievement (CTBS-1992) |
|---|---|---|---|---|---|---|---|
| 4.1 | Male | 9.6 | Caucasian | Reduced | SLD | 100 K-ABC | 88 |
| 4.2 | Female | 8.4 | Caucasian | Free | SLD | 88 K-ABC | 42 |
| 4.3 | Female | 9.5 | Caucasian | Free | SLD | 95 K-ABC | Not Available |
| Classroom 4 Other Group Members N=16 | Female<br>7<br>Male<br>9<br>Total<br>16 | Median<br>8.3<br>Range<br>7.8-8.11 | African-American<br>4<br>Caucasian<br>12 | Free/ Reduced Lunch<br>8<br>Pay<br>8 | Ch. 1<br>2 |  | N=15<br>Median<br>65<br>Range<br>33-98 |

Note: CTBS: Comprehensive Test of Basic Skills (National Percentile)
SLD: Specific Learning Disability
K-ABC: Kaufman Assessment Battery for Children

Table 13

Description of Subjects--Classroom 5

| Total in Group | Permission Slips | Number Excluded | Number Described |
|---|---|---|---|
| 25 | 19 | 0 | 19 |

| Subject | Gender | Age | Ethnicity | Lunch Program | Status | IQ | Mathematics Achievement (CTBS-1992) |
|---|---|---|---|---|---|---|---|
| 5.1 | Female | 9.3 | Caucasian | Free | SLD | 100 K-ABC | 98 |
| Classroom 5<br>Other Group Members<br>N=18 | Female<br>11<br>Male<br>7<br>Total<br>18 | Median<br>8.4<br>Range<br>7.8-8.11 | African-American<br>1<br>Caucasian<br>17 | Free/ Reduced Lunch<br>3<br>Pay<br>15 | Ch. 1<br>2 | | N=18<br>Median<br>87.5<br>Range<br>12-99 |

Note:
CTBS: Comprehensive Test of Basic Skills (National Percentile)
SLD: Specific Learning Disability
K-ABC: Kaufman Assessment Battery for Children

Table 14

Description of Subjects--Classroom 6

| Total in Group | Permission Slips | Number Excluded | Number Described |
|---|---|---|---|
| 26 | 17 | 1 (screening) | 16 |

| Subject | Gender | Age | Ethnicity | Lunch Program | Status | IQ | Mathematics Achievement (CTBS-1992) |
|---|---|---|---|---|---|---|---|
| 6.1 | Male | 8.9 | Caucasian | Pay | SLD | 115 K-ABC | 88 |
| 6.2 | Male | 8.6 | Caucasian | Free | SLD | 96 K-ABC | 46 |
| 6.3 | Male | 8.9 | Caucasian | Free | SLD | 93 K-ABC | 90 |
| Classroom 6 Other Group Members N=13 | Female<br>6<br>Male<br>7<br>Total<br>13 | Median<br>8.3<br>Range<br>7.8-9.4 | African-American<br>2<br>Caucasian<br>11 | Free/ Reduced Lunch<br>7<br>Pay<br>6 | Ch. 1<br>2 |  | N=13<br>Median<br>67<br>Range<br>2-98 |

Note: CTBS: Comprehensive Test of Basic Skills (National Percentile)
SLD: Specific Learning Disability
K-ABC: Kaufman Assessment Battery for Children

Assessment Battery for Children (K-ABC). The K-ABC has a mean of 100 and standard deviation of 15 (Salvia & Ysseldyke, 1985). Mathematics achievement scores were obtained from the Total Math section of the Comprehensive Test of Basic Skills (CTBS) administered a few weeks prior to this investigation. Socioeconomic status is indicated by eligibility for the assisted lunch program available to eligible families in Union County.

### Classroom 1

In Classroom 1, the instructional group included 25 students. Permission to report results was obtained for 23 students. Following screening for tool and prerequisite skills, 4 students were excluded from data analysis. The remaining 19 students included 16 normally achieving students and 3 exceptional students described as Subjects 1.1, 1.2, and 1.3.

Within the normally achieving group, there were 8 female subjects (50%) and 8 male subjects (50%). Subjects ranged in age from 7 years, 8 months to 8 years, 7 months. The median age was 8 years, 1 month. Four subjects (25%) were African-American. Twelve subjects (75%) were Caucasian. Eight subjects (50%) qualified for free or reduced lunch. Four subjects (25%) attended the Chapter 1 program for reading. On the CTBS, national percentile scores for total mathematics ranged from 18 to 98. The median percentile rank was 69 (N=16).

Subject 1.1. Subject 1.1 was a 7-year, 11-month old Caucasian female who received SLD services for 10 hours each week (primarily for reading). The subject was mainstreamed for all other instruction. Testing for special education took place during the summer of 1991. The subject performed in the superior range of intellectual functioning on the K-ABC. The subject was reported to be working below her ability level in both reading and mathematics. On the CTBS, the subject's national percentile score for total mathematics was 13. Subject 1.1 qualified for free lunch assistance.

Subject 1.2. Subject 1.2 was an 8-year, 6-month old Caucasian male who received SLD services for 10 hours each week (primarily for reading). The subject was mainstreamed for all other instruction. Testing for special education took place during 1991. The subject performed in the average range of intellectual functioning on the K-ABC. The subject was reported to be working below his ability level in both reading and mathematics. On the CTBS, the subject's national percentile score for total mathematics was 9. Subject 1.2 qualified for free lunch assistance.

Subject 1.3. Subject 1.3 was an 8-year, 3-month old Caucasian male who received SLD services for 10 hours each week (primarily for reading). The subject was mainstreamed for all other instruction. Testing for special education took place 2 months before this investigation began. The subject performed in the average range of intellectual functioning on the K-ABC. The subject was reported to be working below his ability level in both reading and mathematics. Standardized testing information was not available. Subject 1.3 did not participate in the free or reduced lunch programs.

### Classroom 2

In Classroom 2, the instructional group included 27 students. Permission to report results was obtained for 19 students. No students were excluded after screening for tool and prerequisite skills. The 19 students included 17 normally achieving students and 2 exceptional students described as Subjects 2.1 and 2.2.

Within the normally achieving group, there were 9 female subjects (53%) and 8 male subjects (47%). Subjects ranged in age from 7 years, 8 months to 8 years, 8 months. The median age was 8 years, 4 months. One subject (6%) was African-American. Sixteen subjects (94%) were Caucasian. Five subjects (29%) qualified for free or reduced lunch.

One subject (6%) attended the Chapter 1 program for reading. On the CTBS, national percentile scores for total mathematics ranged from 25 to 99. The median percentile rank was 88 (N=17).

Subject 2.1. Subject 2.1 was an 8-year, 3-month old Caucasian male who qualified for SLD services. The subject was mainstreamed for all academic instruction. Testing for special education took place 3 months before this investigation began. The subject performed in the average range of intellectual functioning on the K-ABC. The subject was reported to be working below his ability level in reading and at his ability level in mathematics. On the CTBS, the subject's national percentile score for total mathematics was 76. Subject 2.1 did not participate in the free or reduced lunch programs.

Subject 2.2. Subject 2.2 was a 9-year, 4-month old Caucasian male who received SLD services for 10 hours each week (primarily for reading). The subject was mainstreamed for all other instruction. Subject 2.2 was repeating second grade. Testing for special education took place during 1990. The subject performed in the slow learner range of intellectual functioning on the K-ABC. The subject was reported to be working at his ability level in reading and below his ability level in mathematics. On the

CTBS, the subject's national percentile score for total mathematics was 17. Subject 2.2 qualified for free lunch assistance.

### Classroom 3

In Classroom 3, the instructional group included 26 students. Permission to report results was obtained for 25 students. Following screening for tool and prerequisite skills, 4 students were excluded from data analysis. One exceptional student was excluded after the baseline phase of monitoring. The remaining 20 students included 19 normally achieving students and 1 exceptional student described as Subject 3.1.

Within the normally achieving group, there were 10 female subjects (53%) and 9 male subjects (47%). Subjects ranged in age from 7 years, 10 months to 8 years, 7 months. The median age was 8 years, 5 months. Three subjects (16%) were African-American. Fifteen subjects (79%) were Caucasian. One subject (5%) was Hispanic. Eleven subjects (58%) qualified for free or reduced lunch. Two subjects (10%) attended the Chapter 1 program for reading. The Hispanic subject attended a remedial program for migrant students. On the CTBS, national percentile scores for total mathematics ranged from 30 to 99. The median percentile rank was 69.5 (N=18).

Subject 3.1. Subject 3.1 was an 8-year, 8-month old Caucasian female who received EH services for 10 hours each week and was mainstreamed for the remainder of the school week. The subject was repeating second grade. Testing for special education took place in 1991. The subject performed in the high average range of intellectual functioning on the K-ABC. The subject was reported to be working below her ability level in both reading and mathematics. On the CTBS, the subject's national percentile score for total mathematics was 90. Subject 3.1 did not participate in the free or reduced lunch programs.

## Classroom 4

In Classroom 4, the instructional group included 26 students. Permission to report results was obtained for 22 students. Following screening for tool and prerequisite skills, 2 students were excluded from data analysis. One exceptional student was excluded after the baseline phase of monitoring. The remaining 19 students included 16 normally achieving students and 3 exceptional students described as Subjects 4.1, 4.2, and 4.3.

Within the normally achieving group, there were 7 female subjects (44%) and 9 male subjects (56%). Subjects ranged in age from 7 years, 8 months to 8 years, 11 months. The median age was 8 years, 3 months. Four subjects (25%)

were African-American. Twelve subjects (75%) were Caucasian. Eight subjects (50%) qualified for free or reduced lunch. Two subjects (12%) attended the Chapter 1 program for reading. On the CTBS, national percentile scores for total mathematics ranged from 33 to 98. The median percentile rank was 65 (N=15).

Subject 4.1. Subject 4.1 was a 9-year, 6-month old Caucasian male who received SLD services for 10 hours each week (primarily for reading). The subject was mainstreamed for all other instruction. Subject 4.1 was repeating second grade. Testing for special education took place during 1990. The subject performed in the average range of intellectual functioning on the K-ABC. The subject was reported to be working below his ability level in reading and above his ability level in mathematics. On the CTBS, the subject’s national percentile score for total mathematics was 88. Subject 4.1 qualified for reduced lunch assistance.

Subject 4.2. Subject 4.2 was an 8-year, 4-month old Caucasian female who received SLD services for 10 hours each week (primarily for reading). The subject was mainstreamed for all other instruction. Testing for special education took place 3 months before this investigation began. The subject performed in the average range of intellectual functioning on the K-ABC. The subject was reported to be working below her ability level in reading and close to her ability level in mathematics. On the CTBS, the subject's national percentile score for total mathematics was 42. Subject 4.2 qualified for free lunch assistance.

*Subject 4.3*. Subject 4.3 was a 9-year, 5-month old Caucasian female who received SLD services for 10 hours each week (primarily for reading). The subject was mainstreamed for all other instruction. Subject 4.3 was repeating second grade. Testing for special education took place in 1991. The subject performed in the average range of intellectual functioning on the K-ABC. The subject was reported to be working below her ability level in both reading and mathematics. Standardized testing information was not available. Subject 4.3 qualified for free lunch assistance.

### Classroom 5

In Classroom 5, the instructional group included 25 students. Permission to report results was obtained for 19 students. No students were excluded after screening for tool and prerequisite skills. The 19 students included 18 normally achieving students and 1 exceptional student described as Subject 5.1.

Within the normally achieving group, there were 11 female subjects (61%) and 7 male subjects (39%). Subjects ranged in age from 7 years, 8 months to 8 years, 11 months. The median age was 8 years, 4 months. One subject (6%) was African-American. Seventeen subjects (94%) were Caucasian. Three subjects (17%) qualified for free or reduced lunch. Two students (11%) attended the Chapter 1 program for reading. On the CTBS, national percentile scores for total mathematics ranged from 12 to 99. The median percentile rank was 87.5 (N=18).

Subject 5.1. Subject 5.1 was a 9-year, 3-month old Caucasian female who received SLD services for 10 hours each week (primarily for reading). The subject was mainstreamed for all other instruction. Subject 5.1 was repeating second grade. Testing for special education took place in 1991. The subject performed in the average range of intellectual functioning on the K-ABC. The subject was reported to be working below her ability level in both reading and mathematics. On the CTBS, the subject's national percentile score for total mathematics was 98. Subject 5.1 qualified for free lunch assistance.

### Classroom 6

In Classroom 6, the instructional group included 26 students. Permission to report results was obtained for 17 students. Following screening for prerequisite skills, 1 student was excluded from data analysis. The remaining 16 students included 13 normally achieving students and 3 exceptional students described as Subjects 6.1, 6.2, and 6.3.

Within the normally achieving group, there were 6 female subjects (46%) and 7 male subjects (54%). Subjects ranged in age from 7 years, 8 months to 9 years, 4 months. The median age was 8 years, 3 months. Two subjects (15%) were African-American. Eleven subjects (85%) were Caucasian. Seven subjects (54%) qualified for free or reduced lunch. Two subjects (15%) attended the Chapter 1 program for reading. On the CTBS, national percentile scores for total mathematics ranged from 2 to 98. The median percentile rank was 67 (N=13).

Subject 6.1. Subject 6.1 was an 8-year, 9-month old Caucasian male who received SLD services for 10 hours each week (primarily for reading). The subject was mainstreamed for all other instruction. Subject 6.1 was repeating second grade. Testing for special education took place during 1990. The subject performed in the high average range of intellectual functioning on the K-ABC. The subject was reported to be working below his ability level in both reading and mathematics. On the CTBS, the subject's national percentile score for total mathematics was 88. Subject 6.1 did not participate in the free or reduced lunch programs.

Subject 6.2. Subject 6.2 was an 8-year, 6-month old Caucasian male who received SLD services for 10 hours each week (primarily for reading). The subject was mainstreamed for all other instruction. Testing for special education took place in 1991. The subject performed in the average range of intellectual functioning on the K-ABC. The subject was reported to be working below his ability level in both reading and mathematics. On the CTBS, the subject's national percentile score for total mathematics was 46. Subject 6.2 qualified for free lunch assistance.

Subject 6.3. Subject 6.3 was an 8-year, 9-month old Caucasian male who received SLD services for 10 hours each week (primarily for reading). The subject was mainstreamed for all other instruction. The student had been retained in kindergarten. Testing for special education took place in 1991. The subject performed in the average range of intellectual functioning on the K-ABC. The subject was reported to be working below his ability level in reading and at his ability level in mathematics. On the CTBS, the subject's national percentile score for total mathematics was 90. Subject 6.3 qualified for free lunch assistance.

## Summary

For this investigation, 13 exceptional subjects in 6 classrooms were monitored individually during large-group mainstream instruction in mathematics. A summary of exceptional subject characteristics is provided (Table 15).

Five of the exceptional subjects were female (38%). Eight of the exceptional subjects were male (62%). Subjects' ages ranged from 7 years, 11 months to 9 years, 6 months. Six subjects (46%) were repeating second grade. One subject (8%) had been retained in kindergarten. All of the subjects were Caucasian (100%). Nine subjects (69%) received free or reduced lunch assistance. Twelve subjects (92%) were classified as SLD. One subject (8%) was labeled as EH. Twelve of the subjects (92%) received special education services for 10 hours each week. One student (8%), classified as SLD, remained in the mainstream classroom for the entire school day.

All subjects had been tested using the K-ABC. IQ scores ranged from 72 to 125. One student's score was in the slow learner range of intellectual functioning (8%). Nine students (69%) performed in the average range. Two students (15%) performed in the high average range. One student (8%) performed in the superior range. On CTBS testing for the 1991-92 school year, national percentile

Table 15

Summary of Exceptional Subject Characteristics

| Subject | Gender | Age | Ethnicity | Lunch Program | Status | IQ | Mathematics Achievement (CTBS-1992) |
|---|---|---|---|---|---|---|---|
| 1.1 | Female | 7.11 | Caucasian | Free | SLD | 125 K-ABC | 13 |
| 1.2 | Male | 8.6 | Caucasian | Free | SLD | 96 K-ABC | 9 |
| 1.3 | Male | 8.3 | Caucasian | Pay | SLD | 91 K-ABC | N/A |
| 2.1 | Male | 8.3 | Caucasian | Pay | SLD | 102 K-ABC | 76 |
| 2.2 | Male | 9.4* | Caucasian | Free | SLD | 72 K-ABC | 17 |
| 3.1 | Female | 8.8* | Caucasian | Pay | EH | 120 K-ABC | 90 |
| 4.1 | Male | 9.6* | Caucasian | Reduced | SLD | 100 K-ABC | 88 |
| 4.2 | Female | 8.4 | Caucasian | Free | SLD | 88 K-ABC | 42 |
| 4.3 | Female | 9.5* | Caucasian | Free | SLD | 95 K-ABC | N/A |

Table 15--Continued

| Subject | Gender | Age | Ethnicity | Lunch Program | Status | IQ | Mathematics Achievement (CTBS-1992) |
|---|---|---|---|---|---|---|---|
| 5.1 | Female | 9.3* | Caucasian | Free | SLD | 100 K-ABC | 98 |
| 6.1 | Male | 8.9* | Caucasian | Pay | SLD | 115 K-ABC | 88 |
| 6.2 | Male | 8.6 | Caucasian | Free | SLD | 96 K-ABC | 46 |
| 6.3 | Male | 8.9* | Caucasian | Free | SLD | 93 K-ABC | 90 |

Note: CTBS: Comprehensive Test of Basic Skills (National Percentile)
SLD: Specific Learning Disability
EH: Emotionally Handicapped
K-ABC: Kaufman Assessment Battery for Children
* Retained
N/A: Information Not Available

scores for total mathematics ranged from 9-98. Five students (38%) scored below the 50th percentile. Six students (46%) scored above the 50th percentile. Testing information was not available for 2 subjects.

## Description of Teachers and Teacher Training

Six female classroom teachers participated in this investigation. The teachers had between 5 and 18 years of teaching experience. All of the teachers had taught second grade for at least 4 years. Other teaching experience was at the elementary level.

The teachers were all certified in elementary education. Two of the teachers were also certified in early childhood education. One teacher was working on additional certification in gifted education. None of the teachers were certified in special education or had taken special education course work.

One training session was conducted to familiarize teachers with the procedures and materials of the Strategic Math Series. The training session was based on procedures previously used at the University of Florida to train teachers in using the Learning Strategies Curriculum materials developed at the University of Kansas Institute for Research in Learning Disabilities (Deshler & Schumaker, 1986).

The training session, lasting approximately 2 hours, occurred within a 1-day workshop on effective teaching procedures. Following an introduction that extended the effective teaching research to mathematics instruction, components of the Strategic Math Series were described and modeled. Each teacher was provided with the Multiplication Facts 0 to 81 (Mercer & Miller, 1992b) manual, all student materials, and one set of manipulative objects to be used during the investigation. As materials were described, teachers had the opportunity for guided practice and feedback on giving scripted directions, using manipulatives, scoring learning sheets, scoring abstract probes, charting student progress, and maintaining student folders. The experimenter met with the teachers on a regular basis and was available to the teachers throughout the investigation.

## Setting

All instruction took place in six second grade classrooms at Lake Butler Elementary School, a public school serving pre kindergarten through fourth grade students in Union County in north central Florida. The total enrollment for the 1991-92 school year was 886 students. Approximately 18% of the total student population qualified for special education services according to state of Florida (exceptional student education) guidelines.

All classrooms were self-contained. Four classrooms were in one wing of the school and shared office and supply space. The other two classrooms were in another wing and physically separated from the four adjoining classrooms as well as from each other. There were no paraprofessionals, volunteers, or interns working with students in any of the classrooms. One teacher had a volunteer who worked in the office area and performed clerical functions.

Instruction took place during regularly scheduled mathematics classes at the point in the year when multiplication was typically introduced (final unit of the year). Once the requirements for the experimental design were met, each teacher scheduled instruction to fit her own classroom routines.

## Instructional Materials

Multiplication Facts 0 to 81 (Mercer & Miller, 1992b), one of the scripted manuals from the Strategic Math Series, was used by six mainstream teachers to teach multiplication to their second grade classes as the final mathematics unit of the school year. The manual provides explicit teaching formats and includes the theory and rationale that guided development of the series; field testing results; description of the instructional phases; description of teaching procedures; 10 scripted lessons using the C-R-A sequence (with pretest and posttest); 11 scripted lessons on solving word problems and increasing rate of computation; directions for playing instructional games with teaching hints and notes; and all student lessons, management materials, scoring keys, and evaluation guidelines. Each section of the manual is described separately. Permission to duplicate materials was obtained from the authors of the series, and samples of student materials described in this section are provided in Appendix C.

### Theory and Rationale

In this section of the manual, the background of the concrete to representational to abstract teaching sequence (C-R-A) is explained. Examples of instructions and activities at each level are provided. This section also provides an overview of the entire Strategic Math Series and the suggested sequence for instruction.

### Results of Field Testing

This part of the manual describes the results of field testing for Multiplication Facts 0 to 81 (Mercer & Miller, 1992b). For 64 students with learning disabilities, the mean posttest score for computation was 92%. For solving word problems, mean posttest scores were 100% for problems without extraneous information, 92% for problems with extraneous information, and 71% on creating word problems. On the average, students increased their ability to solve fact problems by 47% and their speed by 400%. The program was rated better than previous mathematics instruction by 68% of the students, and 100% of the teachers indicated that they would use the program again.

### Instructional Phases

Seven instructional phases are described. Phase 1 is a pretest lesson. In Phase 2, students learn concrete application of multiplication using manipulative objects (lessons 1-3). Application at the representational level occurs in Phase 3 (lessons 4-6) through drawings that replace the manipulative objects. Phase 4 is a single lesson (lesson 7) on using a mnemonic strategy for solving computation problems. Abstract application takes place in Phase 5 (lessons 8-10). During abstract application, students supply known answers immediately or use the mnemonic strategy to figure out unknown answers. Phase 6 is a posttest lesson. Phase 7 covers lessons 11-21 and emphasizes complex word problems and fluency practice.

### Teaching Procedures

The section on teaching procedures provides an explanation and description of the teacher's role in using effective teaching procedures. General procedures that are followed throughout instruction include (a) give an advance organizer, (b) describe and model the process, (c) conduct guided practice, (d) conduct independent practice, (e) conduct problem-solving practice, (f) provide fluency practice, (g) review facts, (h) conduct instructional game practice, and (i) give feedback.

### Scripted Lessons 1-10 (Including Pretest and Posttest)

The pretest lesson is provided to assess students' prior knowledge of multiplication, document the need for instruction, and obtain students' written commitment to learn multiplication. Scripted instruction for lessons 1-10 emphasizes understanding of the multiplication process using concrete (C), representational (R), and abstract (A) procedures. Between the representational and abstract lessons, students learn a mnemonic strategy for independent solution of unknown problems. The mnemonic cue is DRAW, and each letter prompts a student action: D--Discover the sign; R--Read the problem; A--Answer, or draw and check; W--Write the answer.

Lessons include student learning sheets with 10 problems designated for independent practice. Students progress through the C-R-A sequence by demonstrating mastery (80%) on the independent practice component of each lesson. Feedback is provided throughout instruction, and teachers are directed to reteach lessons with scores below 80%. The posttest is the final lesson of the C-R-A sequence, and students are expected to demonstrate mastery of the mathematical operation through a score of 90% prior to beginning lesson 11. For scores below 90%, feedback includes repetition of lesson 10 and the posttest.

Scripted Lessons 11-21

In lessons 11-21, independent practice includes complex word problems. Students learn an additional mnemonic strategy, FAST, to set up word problems: F--Find what you're solving for; A--Ask yourself, "What are the parts of the problem?"; S--Set up the numbers; T--Tie down the sign. Once a problem is set up, it can be solved using the DRAW strategy. On several lessons, subjects are also required to write their own word problems with appropriate labels and accurate solutions. Reteaching is not required for lessons with scores below the 80% mastery criterion.

For increasing rate of computation, students begin solving basic multiplication facts under timed conditions using the Multiplication Minute after selected lessons. Written products are scored in terms of correct and incorrect digits. Mastery is specified as 30 correct digits per minute with 2 or fewer errors (incorrect responses).

### Instructional Games and Helpful Hints

Teachers are provided with supplies and directions for an instructional game found to be highly motivating during field testing. Although an optional activity, teachers are encouraged to use the game for extra practice beginning with lesson 11.

### Management, Scoring, and Evaluation

Student progress (percentage of correct responses and rate of performance) is measured, recorded on an individual progress chart, and used to provide positive and corrective feedback. Teaching scripts, student learning sheets, tests, and probes are included in the manual. All materials needed to manage the scoring and graphic display are provided and explained.

## Experimental Procedures

In all six of the classrooms that participated in this investigation, students move through the basal mathematics series as an intact group. Since a primary purpose of this investigation was to examine the effectiveness of Multiplication Facts 0 to 81 (Mercer & Miller, 1992b) for students with disabilities under regular classroom conditions, the whole-class format was maintained.

The investigation began during the final 8 weeks of the school year. Teachers followed a controlled schedule until all classrooms had completed the second lesson in the instructional sequence. Beginning with lesson 3, teachers scheduled instruction as it fit into their individual classroom circumstances. Except for use of the specific procedures built into the curriculum, no attempt was made to change or interfere with existing classroom conditions. Instruction was discontinued during the last week of school. Throughout the investigation, instruction was organized through the use of student folders. All student materials were kept in the folders and sent home with the students during the last week of school.

Regarding student absences, teachers were asked to maintain the C-R-A sequence specified for the first 10 lessons. This was usually accomplished by working individually with a student on a missed lesson prior to presentation of a new lesson to the entire group. Teachers provided alternate activities for substitutes to use during their own absences.

### Baseline

Rate of computation on basic multiplication facts was monitored continuously for several days prior to instruction as well as across the instructional phases using alternate forms (Appendix D) of the Multiplication Minute provided in the manual. Through scripted instructions, teachers directed students to begin with the first problem, attempt all problems without skipping, and continue working for 1 minute. During baseline, the experimenter scored the probes for frequency (movements per minute) of correct and incorrect responses. Scoring procedures followed the guidelines provided for the Multiplication Minute. Students received no feedback on their performance.

When directed by the experimenter, teachers administered the untimed pretest provided in the manual to document the need for instruction and provide feedback to students. The pretest score served as a baseline measure for untimed performance and became the first recorded measure on individual progress charts. Following feedback, students were asked to make a written commitment to learn multiplication.

### Lessons 1-10 (C-R-A Sequence and DRAW Mnemonic)

Following baseline and pretesting, each teacher taught the first 10 scripted lessons from Multiplication Facts 0 to 81 (Mercer & Miller, 1992b): 3 lessons at the concrete level, 3 lessons at the representational level, 1 lesson on the use of a mnemonic strategy (DRAW) to solve problems, and 3 lessons at the abstract level. The instructional emphasis was on understanding and solving multiplication problems accurately using first manipulatives, then drawings, and finally the DRAW strategy.

The student learning sheets for each lesson were used as directed in the manual. For the beginning of each lesson, teachers provided an advance organizer, described and modeled the process, and conducted guided practice with increasing levels of student involvement. For independent practice (10 problems), students solved specified problems without teacher or peer help. There were no time restrictions on the learning sheets, and students completed the problems at their own rates. As students completed the independence practice problems, teachers graded the lesson and provided feedback. Performance was recorded on students' progress charts as the percentage of correct responses. During the first 10 lessons, independent practice scores below 80% required repetition of the lesson. When repetition was necessary, all attempts were recorded.

To maintain continuous monitoring of students' performance under timed conditions, one form of the abstract probe used during baseline was administered after each lesson. During lessons 1-10, the experimenter scored the probes for frequency of correct and incorrect responses. As during baseline, students received no feedback on their rate performance.

Following lesson 10, teachers administered the posttest. The mastery criterion for moving to lesson 11 was 90% on the posttest. Students who failed to reach the 90% criterion repeated lesson 10 and the posttest. Scores were recorded on students' progress charts.

### Lessons 11-21 (Word Problems and Rate of Computation)

Beginning with lesson 11, the instructional emphasis changed to solving word problems and increasing the rate of computation. During lesson 11, students learned to set up and solve word problems using FAST DRAW.

After lesson 11, teachers were no longer directed to describe and model procedures. Students began independent practice immediately after the advance organizer. Independent practice sections of the learning sheets included word problems with and without extraneous information and blanks for students to create their own multiplication word problems. Each lesson included at least three word problems, and performance was recorded on students' progress charts. By contrast with lessons 1-10, scores lower than the mastery criterion of 80% did not require reteaching or prevent progress to the next lesson.

Rate continued to be monitored with the abstract probe. For lessons 11-21, teachers were trained to count correct and incorrect digits. Rate measures were recorded on students' progress charts, and feedback was provided for both independent practice problems and rate performance. The mastery criterion of 30 correct digits with 2 or fewer errors was used for all subjects in this investigation.

## Measurement Instrumentation

Three measurement instruments were used during this investigation. An experimenter-designed screening instrument based on descriptions of tool and prerequisite skills by Evans, Evans, and Mercer (1986) was used during subject selection (Appendix A). The first section of the instrument was a 1-minute timed probe on the tool skill of writing digits (0-9) from memory. The prerequisite skills of counting to 81 and computing addition facts 0-18 were assessed (untimed) in the second section.

Ten problems designated as independent practice on each student learning sheet (samples provided in Appendix C) were used to measure students' mastery of multiplication concepts throughout instruction. Independent practice problems were untimed, and answers were marked as either correct or incorrect. For each lesson, correct answers were counted and given a percentage score.

Alternate forms of the Multiplication Minute were used to measure students' rate of responding (Appendix D). For all administrations of the probe, students had 1 minute to complete as many problems as possible. Both correct and incorrect digits written during the timing period were counted and recorded as frequency measures.

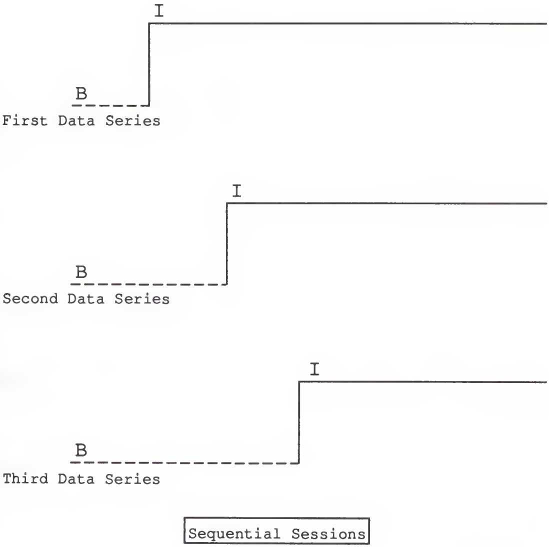
## Experimental Design

Two studies were conducted simultaneously. In each study, a single subject multiple baseline design (Tawney & Gast, 1984) across three functionally independent, yet similar, classrooms was used to demonstrate experimental control and to document the effects of the intervention. The three classrooms in Study I are described as Classrooms 1, 2, and 3. In Study II, the classrooms are described as Classrooms 4, 5, and 6.

The multiple baseline design is characterized by continuous measurement of the dependent variable (target behavior) across several data series prior to introduction of the independent variable (intervention). The independent variable is introduced into one data series when baseline data are stable in all data series. When the target behavior in the data series exposed to the independent variable reaches a criterion level, the independent variable is then introduced into the second data series. The sequence is continued until the independent variable has been introduced into all data series. In a multiple baseline design, experimental control is demonstrated when there is a positive change in both the level and trend of the target behavior in each data series only after the introduction of the independent variable into that series (Tawney & Gast, 1984). An outline of the experimental design is provided in Figure 1.

In this investigation, two dependent variables were monitored. One dependent variable was the percentage of correct responses on untimed independent practice problems included with each lesson of the curriculum. The second dependent variable was the number of correct and incorrect digits written per minute (rate of performance) on an abstract probe of basic multiplication facts. Rate of performance was monitored continuously prior to and throughout instruction.

The independent variable was defined as large-group instruction using the structured curriculum, Multiplication Facts 0 to 81 (Mercer & Miller, 1992b) from the Strategic Math Series. Criterion-level responding for introduction of the independent variable into successive baseline series was 80% mastery on the independent practice problems of the first two lessons in the instructional sequence.



B--- = Baseline phase

(Subjects not exposed to independent variable)

I___ = Intervention phase

(Subjects exposed to independent variable)

*Figure 1*. Outline of experimental design.

### Study I

In Study I, baseline conditions for rate were monitored individually for 6 exceptional subjects (5 SLD, 1 EH) in three mainstream classrooms. When baseline performance was interpretable for all monitored subjects, the intervention was introduced into the first classroom (Classroom 1).

Once the independent variable was introduced into the first classroom, the dependent variable (rate) was continuously monitored under intervention conditions in that classroom while baseline conditions continued in the other two classrooms (Classrooms 2 and 3). Instruction began in Classroom 2 when all exceptional subjects in Classroom 1 completed the learning sheets for the first two lessons with 80% accuracy--the mastery criterion for moving between lessons. Baseline conditions continued in Classroom 3 until exceptional subjects in Classroom 2 completed the first two lessons with 80% accuracy. When exceptional subjects in Classroom 2 reached the 80% criterion on the first two lessons, the intervention was introduced into Classroom 3. Instruction continued as specified in the manual through lesson 21 or until the school year ended.

### Study II

In Study II, baseline conditions for rate were monitored individually for 7 exceptional subjects classified as SLD in three mainstream classrooms. When baseline performance was interpretable for all monitored subjects, the intervention was introduced into the first classroom (Classroom 4).

Once the independent variable was introduced into the first classroom, the dependent variable (rate) was continuously monitored under intervention conditions in that classroom while baseline conditions continued in the other two classrooms (Classrooms 5 and 6). Instruction began in Classroom 5 when all exceptional subjects in Classroom 4 completed the learning sheets for the first two lessons with 80% accuracy--the mastery criterion for moving between lessons. Baseline conditions continued in Classroom 6 until exceptional subjects in Classroom 5 completed the first two lessons with 80% accuracy. When exceptional subjects in Classroom 5 reached the 80% criterion on the first two lessons, the intervention was introduced into Classroom 6. Instruction continued as specified in the manual through lesson 21 or until the school year ended.

An outline of the design for introducing the independent variable into each classroom in Study I is provided in Figure 2. The sequence for introducing the independent variable into each classroom in Study II is shown in Figure 3.

Study I

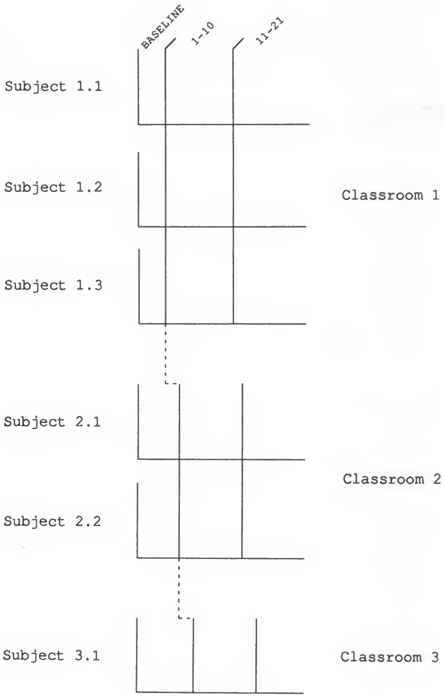


Figure 2. Sequence for introduction of the independent variable into three classrooms.

Study II

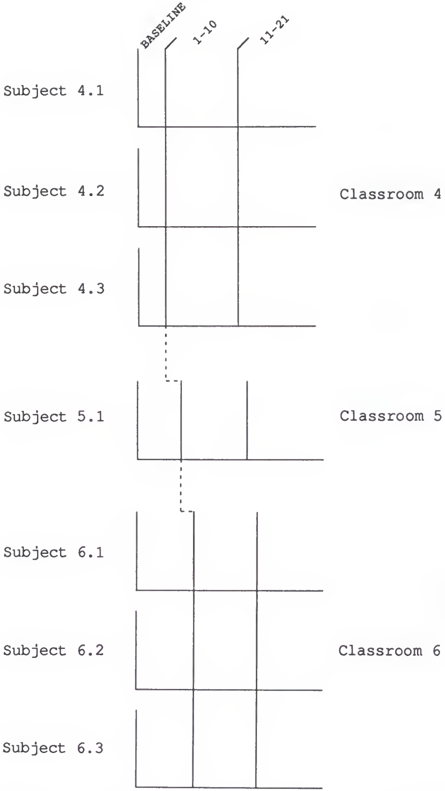


Figure 3. Sequence for introduction of the independent variable into three classrooms.

## Validity and Reliability Procedures

Consideration was given to establishing validity of measurement instruments and reliability of measurement and intervention procedures. The areas of consideration included (a) content validity, (b) social validity, (c) interobserver reliability, and (d) procedural reliability.

### Content Validity

Content validity refers to the match between curriculum content and items on a measurement instrument (Tawney & Gast, 1984). If a measurement instrument and curriculum/instruction are related, then a student's performance on the instrument should adequately represent performance within the curriculum or resulting from instruction (Tindal & Marston, 1990).

Measures of accuracy were taken directly from scores obtained on the independent practice problems included with each lesson in the curriculum used for instruction (Appendix C). The instrument designed to measure rate was a probe containing 54 abstract multiplication problems with products between 0 and 81 (Appendix D). To ensure content validity, the format of the probe was identical to the format of the probe provided with the curriculum. Since students were frequently assessed with the probe, alternate forms were generated by randomly selecting basic facts from 100 possible combinations (Silbert et al., 1990).

### Social Validity

Social validity refers to the social or practical importance of a behavior change (Cooper et al., 1987). Were components of the curriculum manageable under regular classroom conditions? Did students display positive attitudes toward mathematics instruction as well as improvements in performance? Satisfaction with the curriculum and attitudes toward mathematics instruction were assessed through teacher and student surveys adapted from those used during field testing of the *Strategic Math Series*.

*Teacher surveys*. Teachers were asked to evaluate the curriculum components on a 5-point scale (see Appendix E). They were also asked to compare *Multiplication Facts 0 to 81* (Mercer & Miller, 1992b) to other mathematics programs that they had used. Teacher surveys were completed anonymously.

Asked to indicate the degree to which they and their students liked the program, 100% of the teachers marked a rating of *Excellent*. All six teachers described the program as appropriate for use in regular classrooms. Asked to compare *Multiplication Facts 0 to 81* (Mercer & Miller, 1992b) to other mathematics programs, teachers described the curriculum as "excellent," "wonderful--very clear," "much better," and "one of the best." All teachers indicated that they would use the program again, preferably earlier in the school year.

Asked what they liked best about the program, teachers mentioned the concrete lessons and manipulative activities, organization of the materials and lessons, the emphasis on conceptual understanding, students' consistent success, and students' levels of excitement and enthusiasm for learning multiplication. Least liked were long scripts and paperwork. A summary of teachers' assessment of individual lesson components is found in Appendix F.

Student surveys. Students completed their surveys (see Appendix G) during the last week of school. Teachers read the items aloud to avoid misinterpretations. Students indicated whether they liked (happy face) or disliked (sad face) each component described. Results are summarized in Appendix H.

On completed surveys, 97% of the normally achieving subjects (N=97), 100% of the exceptional subjects (N=11), and 100% of students excluded during screening or baseline (N=12) indicated that they liked learning multiplication. Asked to name the best parts of Multiplication Facts 0 to 81 (Mercer & Miller, 1992b), normally achieving subjects most frequently named the Multiplication Minute. Other components frequently mentioned were the DRAW and FAST DRAW strategies, using lines and tallies, working with objects, and learning sheets. Twenty-eight subjects commented that they liked the entire program. Exceptional subjects liked the Multiplication Minute, progress charts, knowing problems faster, and using lines and tallies. Excluded students liked the Multiplication Minute, the FAST DRAW strategy, using objects, using lines and tallies, looking at their progress charts, and receiving feedback.

When asked to name the worst part of the program, normally achieving subjects most frequently mentioned having to write word problem sentences, using lines and tallies, the learning sheets, and the Multiplication Minute. Exceptional subjects mentioned having to count numbers, using the strategy, and the Multiplication Minute. Excluded students mentioned writing word problem sentences, drawing to solve problems, and being timed.

### Interobserver Reliability

Interobserver reliability refers to the level of agreement by independent observers on the occurrence or nonoccurrence of a target behavior. Two target behaviors were monitored throughout this investigation--percentage of correct answers to independent practice problems for each lesson in the curriculum and rate of correct and incorrect responses on timed probes containing basic multiplication facts.

Teachers scored the independent practice problems on student learning sheets. Scoring was similar to teachers' customary manner of scoring mathematics worksheets. Ten independent practice problems were either right or wrong based on answer keys provided in the manual. Throughout the investigation, random lessons were also checked by the experimenter using manual answer keys to verify teachers' use of consistent scoring procedures. No scoring discrepancies were noted.

For the probes used to monitor rate, during baseline and the first 10 lessons, 2 observers independently scored all probes for exceptional subjects. Two observers also independently scored three probes for all other students in each classroom. Once probes became part of instruction and feedback and were kept in student folders (lessons 11-21), five randomly selected probes for exceptional subjects were scored by a second observer. Answer keys were provided for all probe scoring activities.

During baseline and lessons 1-10, there were 5 disagreements out of 636 reliability checks. This resulted in an interobserver agreement score of 99.2% using the formula described by Tawney and Gast (1984):

$$\frac{\text{number of agreements}}{\text{number of agreements} + \text{number of disagreements}} \times 100 = \%\text{ of agreement}$$

During lessons 11-21, there were 2 disagreements out of 65 reliability checks. Use of the same formula resulted in an interobserver agreement score of 96.9%

### Procedural Reliability

Procedural reliability refers to the consistency of implementation of the independent variable (Cooper et al., 1987). An observation form (Appendix I) was used to determine whether teachers were using the procedures outlined for each lesson: (a) give an advance organizer, (b) describe and model, (c) conduct practice (guided, independent, problem-solving, or game), (d) administer the *Multiplication Minute* or *Facts Review*, (e) give feedback.

Three observers familiar with the teaching procedures outlined in the *Multiplication Facts 0 to 81* (Mercer & Miller, 1992b) manual each observed three 10-minute lessons for each teacher and indicated if procedures were being used at timed intervals during the lesson. Observers listened to a cassette tape with beeps at 30-second intervals. When they heard the beep, observers noted which, if any, of the procedures were occurring at that time. Out of a total of 1,080 possible observation intervals, teachers were observed using procedures as specified in the manual for 93.8% of the time (1,014 intervals). Individually, each teacher was observed using procedures as specified in the manual for at least 92% of the observation intervals.

## Data Analysis

In multiple baseline designs, as in all single subject designs, target behaviors are measured directly and repeatedly over time. The resulting data are displayed graphically and interpreted through visual analysis (Parsonson & Baer, 1978; Tawney & Gast, 1984; Wacker, Steege, & Berg, 1988). Cooper et al. (1987) explain that "visual analysis of graphed data is a conservative method of determining the significance of behavior change; only variables able to produce meaningful effects repeatedly are considered significant, and weak and unstable variables are screened out" (p. 139).

To analyze the data for the first two experimental questions of this investigation, both accuracy and fluency measures are displayed for each exceptional subject. For the third experimental question, the individual measures of exceptional subjects are evaluated in terms of the general performance of classmates without disabilities.

### Question 1

To analyze the effects of instruction on the accuracy of responses made by exceptional subjects during independent practice, data were collected for all completed lessons. For each exceptional subject, first and subsequent attempts to reach the mastery criterion required during the C-R-A sequence (lessons 1-10) are displayed. Posttest scores are included with the C-R-A sequence. Scores achieved on lessons that included word problems (lessons 11-21) during which mastery was not required are shown as a separate condition (refer to Figures 2 and 3). Analysis included the percentage of lessons on which the subject reached the mastery criterion and median performance levels during the separate conditions.

### Question 2

To analyze the effects of instruction on fluency, rate of response, "a very sensitive measure of student performance" (Evans et al., 1986, p. 18), was monitored for each exceptional subject. One-minute timings using an abstract probe were administered during baseline and following each lesson. Correct and incorrect written digits were counted and recorded. Lessons 1-10 during which subjects were learning to solve problems accurately using concrete, representational, and abstract procedures and received no feedback on their rates of performance are shown as one instructional condition. Lessons 11-21 during which rate became part of the instructional and feedback cycle are displayed as a separate condition (refer to Figures 2 and 3).

Properties of the data that "demand attention" (Tawney & Gast, 1984, p. 159) were analyzed within and between conditions for each exceptional subject. Within each condition, critical properties included condition length, median response levels for corrects and errors, and the trend of performance. Conditions were also examined for the occurrence of crossover (more correct than incorrect digits) and criterion responding. Between adjacent conditions, critical properties included (a) the number of variables changed between conditions, (b) changes in median level, (c) changes in trend, (d) changes in variability of performance, and (e) percentage of overlap. Changes across similar conditions were analyzed for replication of the effect.

### Question 3

The performance of exceptional subjects was analyzed in terms of the performance of normally achieving subjects. First, individual accuracy and fluency measures were compared to group summary measures (range and median) for each lesson in each classroom. Then, median performance levels during the C-R-A and word problem/fluency practice instructional phases were analyzed for exceptional and normally achieving subjects across both studies.

Within each classroom, exceptional subjects' first attempt scores on the learning sheets for each lesson and on the posttest were compared to the range of scores and median scores obtained by normally achieving subjects receiving the same instruction. The number of correct digits written per minute by exceptional subjects on abstract probes was compared to the range of digits and median number of digits written by normally achieving subjects receiving the same instruction.

For analysis of overall performance during instruction with Multiplication Facts 0 to 81 (Mercer & Miller, 1992b), median performance levels on measures of accuracy and fluency were examined for four instructional phases--concrete phase, representational phase, abstract phase, and word problem/fluency practice phase. Across both studies, the overall performance of exceptional subjects was compared to the overall performance of normally achieving subjects.

# CHAPTER IV
# RESULTS

The purpose of this chapter is to present the results of two studies that investigated the effects of using a scripted, structured curriculum to teach multiplication to large heterogeneous groups of students in elementary classrooms. Six classroom teachers used the procedures and materials of a field-tested curriculum, Multiplication Facts 0 to 81 (Mercer & Miller, 1992b) from the Strategic Math Series, for large-group instruction. Groups ranged in size from 25-27 students and included students with disabilities (exceptional students according to state of Florida guidelines) who were mainstreamed for mathematics instruction.

Subjects for this investigation were exceptional and normally achieving students whose parents or guardians granted permission to report results. Exceptional subjects demonstrated proficiency on tool and prerequisite skills but were neither accurate nor fluent in multiplication. Normally achieving subjects with tool and prerequisite skills were monitored across the six classrooms to establish realistic performance ranges within each classroom.

To examine the effects on exceptional subjects when the structured curriculum was used for large-group instruction, three experimental questions were asked. The first two questions were related to the effects of instruction on the levels of accuracy and fluency reached by exceptional subjects. The third question asked about the performance of exceptional subjects on measures of accuracy and fluency compared to the performance of normally achieving classmates on the same measures.

In each study, a single subject multiple baseline design was used across three independent, yet similar, classrooms to establish experimental control. Classrooms were assigned to either Study I or Study II and given an identifying number. Study I included classrooms designated as Classrooms 1, 2, and 3. Study II included classrooms labeled as Classrooms 4, 5, and 6.

Across the six classrooms, 13 exceptional subjects satisfied the inclusion criteria and were monitored individually throughout the investigation. Within each classroom, exceptional subjects were identified by their classroom number followed by a subject number. Assignment of identifying numbers was random. The numbering system for exceptional subjects within each classroom is outlined in Table 16.

Table 16
Exceptional Subject Numbers Within Each Classroom

| Study I | | | Study II | | |
|---|---|---|---|---|---|
| Classroom 1 | Classroom 2 | Classroom 3 | Classroom 4 | Classroom 5 | Classroom 6 |
| Subject 1.1 | Subject 2.1 | Subject 3.1 | Subject 4.1 | Subject 5.1 | Subject 6.1 |
| Subject 1.2 | Subject 2.2 | | Subject 4.2 | | Subject 6.2 |
| Subject 1.3 | | | Subject 4.3 | | Subject 6.3 |

Across the six classrooms, 99 normally achieving subjects were also monitored individually throughout the investigation. Since only summary information was provided for the normally achieving subjects within each classroom, the classroom number was sufficient for identification purposes.

In this chapter, results are described first for Study I and then for Study II. For each study, results are organized by classroom according to the three experimental questions. Within each classroom, results are described separately for exceptional subjects. Presentation and data analysis procedures for both studies are described in the next section.

## Presentation and Analysis of the Data

Consistent with single subject research methodology, data are displayed graphically for each exceptional subject. An overview of the results is provided in Figure 4. Additional figures and tables are presented with each study. Raw data are found in Appendix J for exceptional subjects and in Appendix K for normally achieving subjects.

To document performance prior to instruction, continuous baseline measures were collected on alternate versions of a timed probe of basic multiplication facts. Following the baseline phase, two dependent variables were monitored concurrently to answer separate experimental questions.

For the experimental question related to accuracy (Question 1), performance was monitored on the untimed independent practice problems of 21 consecutive lessons. Percentage scores are presented as columns on individual equal interval charts. Two conditions were monitored for Question 1. The first instructional condition included lessons 1-10. The second instructional condition included lessons 11-21. Pretest and posttest scores are also provided. Lesson 7 on using the mnemonic DRAW is shown in sequence on the charts, but no practice problems were required for that lesson.

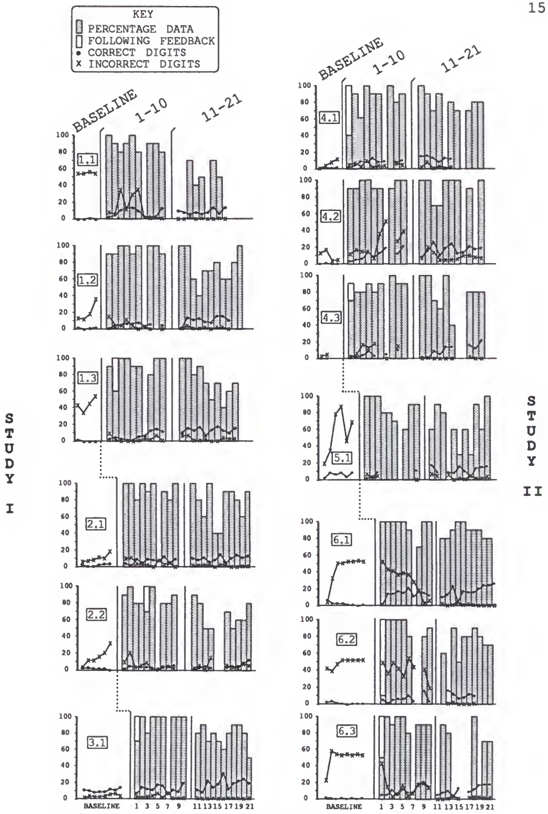


Figure 4. Overview of results in Studies I and II.

Criterion responding on lessons 1-21 was 80%. During lessons 1-10, subjects were required to repeat lessons on which they failed to meet the criterion score. Prior to beginning lesson 11, subjects were expected to reach 90% on the posttest. Repeating the lesson was not required for scores below the criterion during lessons 11-21. Analysis included exceptional subjects' scores on each lesson, the number of repeated lessons, the percentage of lessons with criterion responding, and the median score for each instructional condition. Since a mastery criterion was in effect during the first instructional condition, all attempts to reach the criterion are shown on subjects' charts. However, only first attempt scores were used for computing the percentage of lessons with criterion responding.

For the experimental question related to fluency (Question 2), performance was monitored on alternate versions of a timed probe of basic multiplication facts from baseline through completion of the final lesson in the curriculum. Rate data for Question 2 are recorded as the number of correct and incorrect digits written per minute. For charted data, the number of correct digits is represented by the symbol ●. The number of errors (incorrect digits) is represented by the symbol **X**. Since accuracy scores of 80% on the first two lessons controlled the introduction of the intervention into the second and third classrooms in each study, rate data are superimposed on the accuracy percentages used for answering the first experimental question.

To analyze the effects of instruction on fluency, performance was monitored during three conditions: baseline, lessons 1-10, and lessons 11-21. Length of the baseline condition was based on introduction of the intervention within the requirements of the multiple baseline design.

During baseline, subjects received neither instruction in multiplication nor feedback on their performance on the timed probes. During lessons 1-10, subjects received instruction in solving multiplication problems using concrete, representational, and abstract procedures. The instructional emphasis was on accuracy, and subjects received no feedback on their performance on the timed probes. During lessons 11-21, fluency practice and charting of rate scores became part of the scripted instructional procedure. On probes, subjects continued to solve problems for 1 minute without skipping any problems. They were directed to write the answer immediately if they knew it or to use the DRAW strategy to figure it out.

Criterion responding for fluency (specified in the manual) was 30 correct digits with 2 or fewer errors (incorrect digits) per minute, and subjects received feedback on their progress toward the criterion.

In addition to visual inspection for criterion responding, data were analyzed to determine the effects of the intervention on rate performance (fluency). Analysis required examination of changes both within and between conditions. Correct and incorrect responses were analyzed separately. On tabled summaries, information about correct digits is written first followed by information about incorrect digits. Either a slanted or horizontal line is used to separate types of digits.

Within each condition, properties for analysis included condition length, median performance levels, trend of performance, occurrence of crossover, and attainment of criterion responding. Condition length was defined as the number of plotted data points in each condition. Median scores were calculated for both correct and incorrect responses. Computer-generated curves fitted to the plotted data points using a linear least squares equation were used for analysis of performance trends within each condition. Trend direction was described as accelerating (upward slope), decelerating (downward slope), or level (flat line). Crossover was defined as the point at which subjects made more correct than incorrect responses and maintained the pattern for the rest of instruction. Criterion responding was based on 30 correct digits per minute with 2 or fewer errors.

Critical properties between conditions included the number of variables changed, changes in median level, changes in trend, changes in variability, and percentage of overlap between adjacent conditions. Between conditions, only one variable was changed. From baseline to the first 10 lessons, introduction of the intervention was the only change. Between instructional conditions, the change was a shift in the instructional focus from accuracy to fluency.

Median scores were used to analyze level changes in correct and incorrect responding between conditions. On tabled summaries, increases were labeled with a plus (+) sign, and decreases received a minus (-) sign.

Changes in trend were analyzed by visually comparing the direction of trend lines between adjacent conditions. Variability was defined as the fluctuation of performance around the trend lines of each data path. Fluctuation in each condition was graphically indicated with frequency envelopes that enclosed the data within borders parallel to the trend lines (Tindal & Marston, 1990). Changes in variability between conditions were then visually analyzed and described as increases (+), decreases (-), or no change (=) in performance fluctuation between conditions.

The percentage of overlap was computed to document changes in performance prior to and following instruction. The percentage of overlap was established by dividing the number of data points in the first instructional condition that were within the baseline range by the total number of data points in the condition and multiplying by 100 (Tawney & Gast, 1984). Tindal and Marston (1990) explain that, although no firm criteria have been established, "treatment effects are inversely related to overlap" (p. 385). Across the single subject studies reviewed in Chapter II, the median percentage of overlap was 29% for correct digits and 20% for incorrect digits. Although this investigation was conducted under less controlled classroom conditions than the studies previously reviewed, evidence of treatment effect was evaluated using those median percentages.

In addition to the within and between condition analyses, the overall effect of instruction on fluency was calculated using median scores from baseline and the final condition. The difference between the final condition and baseline was divided by the baseline value and multiplied by 100 to yield the percent of change in correct and incorrect responding. Plus and minus signs were used to indicate the direction of change.

The third question regarding the performance of exceptional subjects with respect to normally achieving subjects in the same instructional group required both individual subject data and summary measures for groups of normally achieving subjects. Relevant data included both percentage of correct responses on lessons 1-21 and the posttest lesson (accuracy) and the number of correct digits written per minute on timed probes (fluency).

Throughout the investigation, performance on both dependent variables was monitored for normally achieving subjects in each classroom, and ranges were established on all measures. By classroom, exceptional subjects' scores that were within the range were divided by the total number of measures and multiplied by 100 to determine the percentage of within-range scores. Within-range performance on 80% of the measures was the criterion for use of the curriculum as a mainstream intervention for large-group instruction in multiplication. Exceptional subjects' scores were also compared to group median scores to establish the percentage of scores that were equal to or above the group median score.

For analyses related to the experimental questions, the baseline condition is referred to as Condition A. Lessons 1-10 are described as Condition B, and lessons 11-21 are designated as Condition C. The same labeling procedure is followed on all figures and tables.

## Study I

For Study I, results were analyzed for 6 exceptional subjects and 52 normally achieving subjects across 3 second grade classrooms. Figures and tables for the three experimental question are grouped together following the analysis of results for each question.

### Question 1

When classroom teachers use _Multiplication Facts 0 to 81_ (Mercer & Miller, 1992b) from the _Strategic Math Series_ for large-group instruction in multiplication, what are the effects on accuracy (in terms of criterion responding) for students with disabilities?

Percentage scores on each lesson are displayed in Figure 5 for subjects in Classroom 1, Figure 6 for subjects in Classroom 2, and Figure 7 for the subject in Classroom 3. Median percentage scores are presented in Figure 8. Results for Question 1 are summarized in Table 17.

_Classroom 1--Subject 1.1_. In Classroom 1 (Figure 5), the first classroom to begin instruction, Subject 1.1

scored 15% on the pretest. The subject completed the entire C-R-A sequence during Condition B and reached or exceeded the 80% criterion on all lessons. On the posttest, the subject reached 90% on the first attempt. The increase from pretest to posttest was 75 percentage points. The subject did not complete the learning sheets for several lessons (11, 12, 16, 19, 20, 21) during Condition C. For completed lessons, scores ranged from 40% to 70%. The subject's median performance during Condition B was 90% and during Condition C was 50% (see Figure 8).

Classroom 1--Subject 1.2. Subject 1.2 scored 10% on the pretest. The subject completed the entire C-R-A sequence during Condition B and scored above the 80% criterion on all lessons. The subject's first score on the posttest was 70%, an increase of 60 percentage points from the pretest. Following feedback, the subject reached the posttest criterion score. The subject completed all lessons of Condition C. Scores ranged from 40% to 100%. Median performance levels were 100% for Condition B and 70% for Condition C (see Figure 8).

Classroom 1--Subject 1.3. Subject 1.3 scored 10% on the pretest. The subject completed the entire C-R-A sequence during Condition B. Lesson 2 was repeated. On lesson 2, the subject's first score was 60%. Following reteaching, the subject reached 100%. The subject scored 65% on the posttest, an increase of 55 percentage points from the pretest score. The subject was absent following the posttest and began lesson 11 without reaching the posttest criterion. During Condition C, the subject completed all lessons except lesson 21. Scores ranged from 40% to 100%. The median score for Condition B was 90% and for Condition C was 70% (see Figure 8).

Classroom 2--Subject 2.1. In Classroom 2 (Figure 6), the second classroom to begin instruction, Subject 2.1 received 30% on the pretest. The subject completed the entire C-R-A sequence during Condition B and reached or exceeded the 80% criterion on all lessons. Two attempts were required to reach the posttest criterion. The first attempt produced a score of 65%, an increase of 35 percentage points from the pretest score. Following feedback, the subject reached 90%. The subject completed all lessons of Condition C. Scores ranged from 40% to 100%. The subject's median performance was 100% during Condition B and 80% during Condition C (see Figure 8).

Classroom 2--Subject 2.2. Subject 2.2 scored 5% on the pretest. The subject completed the entire C-R-A sequence during Condition B. Lesson 5 was the only lesson that had to be repeated. The subject initially scored 70%

on lesson 5. Following reteaching, the subject scored 100%. The subject's first attempt on the posttest produced a score of 30%, an increase of 25 percentage points from the pretest. Following feedback, the subject scored 90%. Subject 2.2 completed all but two (15 and 16) of the lessons in Condition C. Scores ranged from 50% to 90%. The median performance for Subject 2.2 was 80% for Condition B and 60% for Condition C (see Figure 8).

_Classroom 3--Subject 3.1_. In Classroom 3 (Figure 7), the third classroom to begin instruction, Subject 3.1 scored 50% on the pretest. The subject completed the entire C-R-A sequence. The subject's initial score on lesson 1 was 70%. Following reteaching, the subject scored 100%. All other scores were at or above the criterion during Condition B. On the posttest, Subject 3.1 scored 80%, an increase of 30 percentage points from the pretest score. Following feedback, the subject's score was 100%. During Condition C, the subject completed all lessons, and scores ranged from 50% to 90%. The subject's median score for Condition B was 100% and for Condition C was 80% (see Figure 8).

_Summary of results for Question 1_. The results for Question 1 are summarized in Table 17. Pretest scores ranged from 5% to 50% and confirmed the need for instruction in multiplication. During Condition B, all subjects completed the entire C-R-A sequence. No subject repeated more than one lesson during Condition B. Subjects reached the 80% criterion on the first attempt on at least 89% of the lessons, and median scores ranged from 80% to 100%.

All subjects improved from pretest to posttest. The amount of improvement ranged from 25 to 75 percentage points. Initial posttest scores ranged from 30% to 90%. One subject (1.1) reached the 90% posttest criterion on the first attempt. Following feedback and reteaching of lesson 10, 4 other subjects (1.2, 2.1, 2.2, and 3.1) reached the 90% criterion. One subject (1.3) began lesson 11 without reaching criterion on the posttest.

With the introduction of complex word problems to the independent practice section of lessons in Condition C, scores were lower than in the previous condition in all three classrooms. Subjects achieved criterion responding on fewer lessons in Condition C. Five subjects (1.2, 1.3, 2.1, 2.2, and 3.1) reached the criterion score on some lessons while 1 subject (1.1) failed to reach the criterion on any completed lesson. Median scores ranged from 50% to 80%.

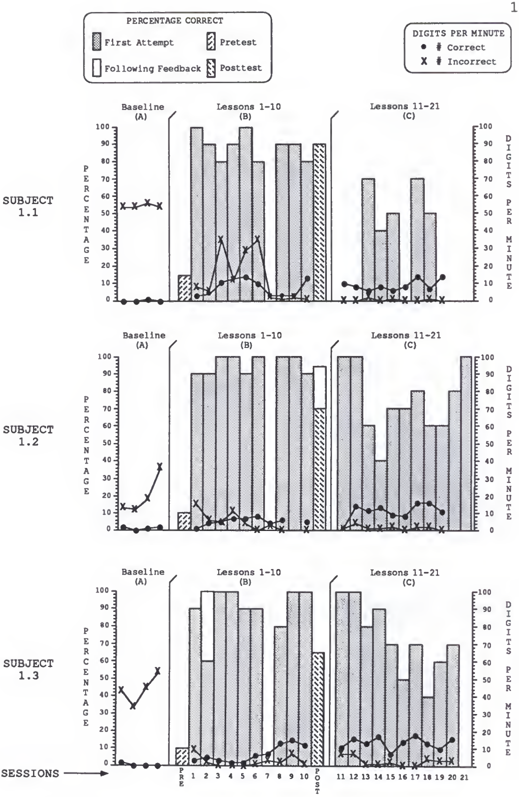


Figure 5. Percentage and rate data for exceptional subjects in Classroom 1.

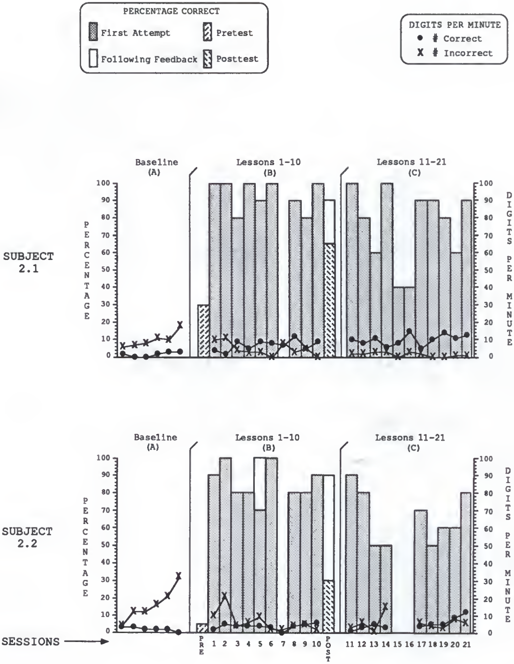


Figure 6. Percentage and rate data for exceptional subjects in Classroom 2.

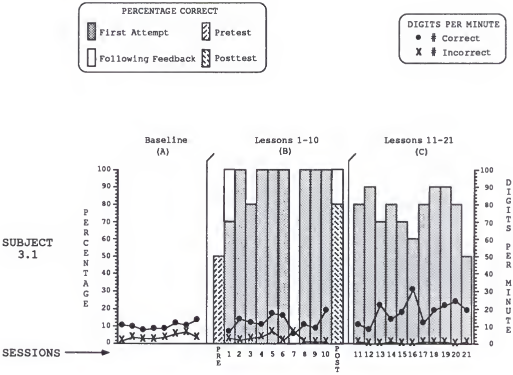


Figure 7. Percentage and rate data for the exceptional subject in Classroom 3.

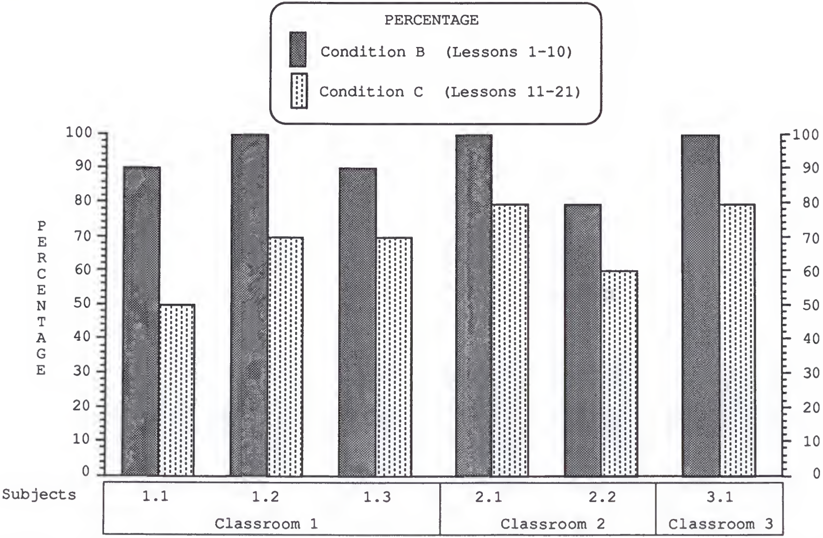


Figure 8. Median percentage scores for exceptional subjects in Classrooms 1, 2, and 3.

Table 17

Summary of Results for Question 1 in Study I

| S | Pretest | Posttest | Increase Pre-Post* | # Lessons Repeated (B) | Lessons at Criterion (B)* | Median % (B) | Lessons at Criterion (C) | Median % (C) |
|---|---|---|---|---|---|---|---|---|
| 1.1 | 15% | 90% | 75% | 0 of 9 | 100% | 90% | 0% | 50% |
| 1.2 | 10% | 70% 95% | 60% | 0 of 9 | 100% | 100% | 45% | 70% |
| 1.3 | 10% | 65% | 55% | 1 of 9 | 89% | 90% | 40% | 70% |
| 2.1 | 30% | 65% 90% | 35% | 0 of 9 | 100% | 100% | 64% | 80% |
| 2.2 | 5% | 30% 90% | 25% | 1 of 9 | 89% | 80% | 33% | 60% |
| 3.1 | 50% | 80% 100% | 30% | 1 of 9 | 89% | 100% | 64% | 80% |

S = Subject * = first attempt scores only

(B) = Condition B (lessons 1-10) (C) = Condition C (lessons 11-21)

Note: Broken line indicates exceptional subjects by classroom.
Posttest column shows all attempts to reach the 90% criterion score.

## Question 2

When classroom teachers use Multiplication Facts 0 to 81 (Mercer & Miller, 1992b) from the Strategic Math Series for large-group instruction in multiplication, what are the effects on fluency (in terms of criterion responding) for students with disabilities?

Individual results are displayed by classroom (refer again to Figures 5, 6, and 7). Computer-generated trend lines for each exceptional subject are presented by classroom in Figures 9, 10, and 11. Frequency envelopes for visual analysis of variability are shown in Figure 12. Ranges and median performance levels are provided in Table 18. A summary of within condition analyses is provided in Table 19. Between condition analyses are summarized in Table 20. Table 21 contains the overall percent of change in median rate scores.

After four consecutive baseline measures, it was evident that performance was unchanged or deteriorating for the exceptional subjects in all three classrooms. Consequently, the intervention was introduced into Classroom 1. Two days were required for exceptional subjects to reach criterion responding on the first two lessons in Classroom 1. During those 2 days, Classrooms 2 and 3 remained under baseline conditions. The intervention was then introduced into Classroom 2 while subjects in Classroom 3 continued baseline monitoring. After the exceptional subjects in Classroom 2 reached criterion responding on the first two lessons (2 days), the intervention was introduced into Classroom 3. During the extended baselines in Classrooms 2 and 3, overall performance did not improve for any subject.

Classroom 1--Subject 1.1. Within condition analysis for Subject 1.1 included 4 measures during Condition A, 10 measures during Condition B, and 9 measures in Condition C. With correct digits recorded over incorrect digits, median performance levels were 0/54 during Condition A, 7/7 for Condition B, and 8/0 during Condition C (Table 18).

Trend lines for within condition analysis are shown in Figure 9. During Condition A, correct responding was low and level while incorrect responding was high and level. During Condition B, correct responding remained level while incorrect responding decelerated. During Condition C, correct responding showed a slight upward trend while incorrect responding stabilized at a low level.

Crossover was maintained following lesson 7. With a high score of 14 correct digits (lessons 5, 17, 19), the subject failed to meet the criterion of 30 for correct responding. Consistently making two or fewer errors from lesson 7 on, the subject did meet the criterion for incorrect digits.

Examination of Table 18 shows that Subject 1.1 increased the median level of corrects by 7 digits between Conditions A and B and by 1 digit between Conditions B and C. Between Conditions A and B, the median level for incorrects dropped 47 digits. A further reduction of 7 incorrect digits occurred between Conditions B and C.

Changes in trend between conditions are shown for Subject 1.1 in Figure 9. Correct responding was low and level during Condition A, remained stable at a slightly higher level during Condition B, and accelerated slightly during Condition C. Incorrect responding was high and level during Condition A, dropped to a lower level and decelerated during Condition B, and stabilized at a low level during Condition C.

Visual inspection of frequency envelopes that encompass all of the data points within each condition (Figure 12) indicates that variability increased for both correct and incorrect digits between Conditions A and B and decreased for both types of digits between Conditions B and C. There were no overlapping data points between Conditions A and B for either correct or incorrect responses.

Classroom 1--Subject 1.2. Within condition analysis for Subject 1.2 included four measures during Condition A, nine measures during Condition B, and nine measures in Condition C. With correct digits recorded over incorrect digits, median performance levels were 1.5/15.5 during Condition A, 5/4 in Condition B, and 11/1 during Condition C (Table 18).

Trend lines for within condition analysis are shown in Figure 9. During Condition A, correct responding remained level while incorrect responding accelerated rapidly. In Condition B, trends showed accelerating correct and decelerating incorrect responses. During Condition C, correct responding continued an upward trend while incorrect responding continued to decelerate. Crossover was maintained following lesson 12. With a high score of 16 correct digits (lessons 17 & 18), the subject failed to meet the criterion of 30 for correct responding. Consistently making two or fewer errors after lesson 12, the subject met the criterion for incorrect digits.

Inspection of Table 18 shows that Subject 1.2 increased the median level of corrects by 3.5 digits between Conditions A and B and by 6 digits between Conditions B and C. The median level for incorrect digits decreased by 11.5 digits between Conditions A and B and decreased again by 3 digits between Conditions B and C.

Changes in trend between conditions are shown for Subject 1.2 in Figure 9. Correct responding was low and level during Condition A, accelerated slightly during Condition B, and continued to accelerate more steeply during Condition C. Incorrect responding accelerated during Condition A, made an abrupt deceleration during Condition B, and continued a slight downward trend during Condition C.

Visual inspection of frequency envelopes that enclose all of the data points within each condition (Figure 12) reveals that variability increased for correct responses and decreased for incorrect responses between Conditions A and B. The pattern was repeated between Conditions B and C. For both correct and incorrect responses, only 11% of the data points overlapped Conditions A and B.

Classroom 1-Subject 1.3. Within condition analysis for Subject 1.3 included 4 measures during Condition A, 10 measures during Condition B and 10 measures in Condition C. With correct digits recorded over incorrect digits, median performance levels were 0/44 during Condition A, 5.5/1.5 in Condition B, and 13.5/2.5 in Condition C (Table 18).

Trend lines for each condition are shown in Figure 9. During Condition A, correct responding showed a slight downward trend while incorrect responding accelerated rapidly. During Condition B, trends showed accelerating correct and decelerating incorrect responses. During Condition C, correct responding leveled off while incorrect responding continued a downward trend. Crossover was maintained following lesson 2. With a high score of 18 correct digits (lesson 17), the subject failed to meet the criterion of 30 for correct responding. With more than two errors on 5 of the 10 completed lessons in Condition C, Subject 1.3 also failed to meet the criterion for incorrect responding.

Median performance levels on Table 18 show that Subject 1.3 increased the number of correct responses by 5.5 digits between Conditions A and B and by 8 more digits between Conditions B and C. Median levels of incorrect digits decreased by 42.5 digits between Conditions A and B. A 1-digit increase in the median number of incorrect digits occurred between Conditions B an C.

Changes in trend between conditions are shown for Subject 1.3 in Figure 9. Correct responding decelerated during Condition A, accelerated during Condition B, and leveled off during Condition C. Incorrect responding, accelerating during Condition A, dropped substantially and took a downward turn during Condition B, then returned to a slightly higher level but continued to decelerate during Condition C.

Visual inspection of frequency envelopes that enclose all of the data points within each condition (Figure 12) indicates that variability increased for correct digits and decreased for incorrect digits between Conditions A and B as well as between Conditions B and C. The percentage of overlapping data points between Conditions A and B was 20% for correct digits and 0% for incorrect digits.

Classroom 2--Subject 2.1. Within condition analysis for Subject 2.1 included 6 measures during Condition A, 10 measures during Condition B, and 11 measures in Condition C. With correct digits recorded over incorrect digits, median performance levels were 2/9 during Condition A, 7.5/3.5 in Condition B, and 10/2 for Condition C (Table 18).

Trend lines for each condition are shown in Figure 10. During Condition A, both correct and incorrect responding had accelerating trends. The effect was slightly more pronounced for incorrect responding. During Condition B, trends showed accelerating correct and decelerating incorrect responses. During Condition C, trends continued to show slight improvement. Crossover was maintained following lesson 10. With a high score of 15 correct digits (lesson 16), the subject failed to meet the criterion of 30 for correct responding. With two or fewer errors on 8 of the 11 completed lessons in Condition C, Subject 2.1 met the criterion for incorrect digits.

Median levels (Table 18) increased for Subject 2.1 by 5.5 digits between Conditions A and B and by 2.5 digits between Condition B and C. The median number of incorrect digits decreased by 5.5 digits between Conditions A and B and by another 1.5 digits between Conditions B and C.

Changes in trend between conditions are shown for Subject 2.1 in Figure 10. Correct responding was accelerating during all three conditions. Incorrect responding accelerated faster than correct responding during Condition A, decelerated during Condition B, and continued to decelerate during Condition C.

Visual inspection of frequency envelopes that encompass all of the data points within each condition (Figure 12) indicates that variability increased for correct responses and remained the same for incorrect responses between Conditions A and B. Between Conditions B and C, variability remained the same for correct responses and decreased for incorrect responses. The percentage of overlapping data points between Conditions A and B was 10% for correct digits and 30% for incorrect digits.

Classroom 2--Subject 2.2. Within condition analysis for Subject 2.2 included 6 measures during Condition A, 10 measures during Condition B, and 9 measures in Condition C. With correct digits recorded over incorrect digits, median performance levels were 2/14 during Condition A, 4/4.5 for Condition B, and 5/6 in Condition C (Table 18).

Trend lines for each condition are shown in Figure 10. During Condition A, correct responses decelerated slightly while incorrect responses showed a rapidly accelerating trend. During Condition B, trends showed accelerating correct and decelerating incorrect responses. During Condition C, correct responding dropped lower but resumed a slight upward trend. Incorrect responding returned to a higher level and stabilized. Crossover was not maintained until lesson 18. With a high score of 12 correct digits on lesson 21, the subject failed to meet the criterion of 30 for correct responding. With more than two errors on eight of the nine lessons completed during Condition C, Subject 2.2 also failed to meet the criterion for incorrect responding.

Examination of Table 18 shows that Subject 2.2 increased the median number of corrects by 2 digits between

Conditions A and B and again by 1 digit between Conditions B and C. The median number of incorrects was reduced by 9.5 digits between Conditions A and B. Incorrects increased by 2.5 digits between Conditions B and C.

Changes in trend between conditions are shown for Subject 2.2 in Figure 10. Correct responding, decelerating during Condition A, accelerated during Condition B, then dropped to a lower level but continued to accelerate during Condition C. Incorrect responding began at a low level but accelerated rapidly during Condition A, took an abrupt downward turn during Condition B, and leveled off above the criterion during Condition C.

Visual inspection of frequency envelopes that enclose all of the data points within each condition (Figure 12) confirms that variability increased for both correct and incorrect responses between Conditions A and B. Between Conditions B and C, variability remained the same for correct responses and decreased slightly for incorrect responses. The percentage of overlapping data points between Conditions A and B was 30% for correct responses and 70% for incorrect responses.

Classroom 3--Subject 3.1. Within condition analysis for Subject 3.1 included 8 measures during the baseline condition, 10 measures during Condition B, and 11 measures in Condition C. With correct digits recorded over incorrect digits, median performance levels were 10.5/4 during Condition A, 11.5/2.5 during Condition B, and 19/1 for Condition C (Table 18).

Trend lines for each condition are shown in Figure 11. During Condition A, responses indicated some prior knowledge of multiplication (correct responses higher than incorrect responses). However. the trend for incorrect responses was accelerating with about the same slope as the trend for correct responses. During Condition B, correct responses accelerated slightly while incorrect responses took a downward turn. During Condition C, correct responses showed an accelerating trend while incorrect responses stabilized at a low level. The subject demonstrated crossover on the first day of Condition A and maintained it except for one lesson (lesson 7) throughout the rest of instruction. With a high score of 31 correct digits (lesson 16), the subject did meet the criterion of 30 for correct responding on a single day. With two or fewer errors from lesson 8 on, Subject 3.1 also met the criterion for incorrect responding.

Examination of Table 18 shows an increase of 1 correct digit in the median levels of Conditions A and B. Between Conditions B and C, the median level of corrects increased by 7.5 digits. Incorrect responses decreased by 1.5 digits between Conditions A and B as well as between Conditions B and C.

Changes in trend between conditions are shown for Subject 3.1 in Figure 11. Correct responding accelerated during Condition A, continued slight acceleration during Condition B, and accelerated more rapidly during Condition C. Incorrect responding accelerated during Condition A, decelerated during Condition B, and stabilized at a low level during Condition C.

Visual inspection of frequency envelopes that enclose all of the data points within each condition (Figure 12) indicates that variability increased for both correct and incorrect responses between Conditions A and B. Between Conditions B and C, variability increased for correct responses and decreased for incorrect responses. The percentage of overlapping data points between Conditions A and B was 50% for correct responses and 70% for incorrect responses.

Within condition summary for Question 2 (Table 19). Baseline (Condition A) included four measures in Classroom 1, six measures in Classroom 2, and eight measures in Classroom 3. There were at least nine measures for all subjects during both of the other two conditions (B and C).

Only 1 subject (3.1) began the investigation with the median of correct responses higher than the median of incorrect responses (refer again to Table 18). All other subjects had more incorrect than correct responses during Condition A. Three subjects (1.2, 1.3, and 2.1) increased the level of corrects above the level of incorrects during Condition B and maintained that pattern for Condition C. Subject 1.1 had equal levels of correct and incorrect responses during Condition B; for that subject, corrects increased above incorrects during Condition C. Only Subject 2.2 still had a higher median level of incorrect responses during Condition C.

During Condition A, Subject 1.1 showed level trends for both correct and incorrect responses. Subject 1.2 had a level trend for corrects, but incorrects were accelerating. Subjects 1.3 and 2.2 showed clearly worsening trends with decelerating corrects and accelerating incorrects. Subjects 2.1 and 3.1 had accelerating trends for both corrects and incorrects. In Condition B, 5 subjects (1.2, 1.3, 2.1, 2.2, and 3.1) showed accelerating corrects and decelerating incorrects. Subject 1.1 had a level trend for corrects with a decelerating trend for incorrects. In Condition C, trends for corrects were either accelerating (Subjects 1.1, 1.2, 2.1, 2.2, and 3.1) or level (Subject 1.3). Trends for incorrects were either decelerating (Subjects 1.2, 1.3, and 2.1) or level (Subjects 1.1, 2.2, and 3.1).

Crossover was demonstrated at different points during instruction. Subject 1.3 showed the change early in Condition B (lesson 2--concrete phase). The effect was also observed during Condition B for Subjects 1.1 (lesson 7--mnemonic phase) and 2.1 (lesson 10--abstract phase). Subject 1.2 showed crossover early in Condition C (lesson 12--fluency practice and word problems). Subject 2.2 finally increased the number of correct digits above the number of incorrect digits at lesson 18 during fluency practice and word problems. Subject 3.1 began the investigation with a crossover pattern of responding.

Only the subject who demonstrated prior knowledge of multiplication (3.1) reached the criterion of 30 for correct responding on a single day. Four subjects (1.1, 1.2, 2.1, and 3.1) achieved the criterion level of two or fewer errors for incorrect responding.

Between condition summary for Question 2 (Table 20). Analysis of changes in median scores reveals that all 6 subjects increased the number of correct digits and reduced the number of incorrect digits between Conditions A and B. Between Conditions B and C, all subjects increased their correct responses again. Four subjects (1.1, 1.2, 2.1, and

3.1) reduced their incorrect responses while 2 subjects (1.3 and 2.2) showed slight increases for incorrect digits.

In general, trends showed low or stable levels of correct responding with accelerating or high stable incorrect responding during Condition A. When the intervention was introduced into each classroom (Condition B), performance improved for all exceptional subjects. Either incorrects turned downward (Subjects 1.1, 2.1, and 3.1) or both corrects and incorrects changed directions (Subjects 1.2, 1.3 and 2.2). When fluency practice and feedback on the probes became part of instruction (Condition C), Subjects 1.1, 1.2, 2.1, and 3.1 showed some improvement in the number of correct responses and stabilized the number of incorrects below the criterion level. Subject 1.3 showed no improvement in corrects, but incorrects were decelerating slowly. Subject 2.2 showed an accelerating trend for corrects, but incorrects were not decreasing. In general, the most change occurred for all subjects between Conditions A and B.

Comparison of performance fluctuation around trend lines indicates that variability in correct responding increased for all subjects between Conditions A and B. Variability for incorrect responding increased for 3 subjects (1.1, 2.2, and 3.1), decreased for 2 subjects (1.2

and 1.3), and remained the same for Subject 2.1. Between Conditions B and C, variability of correct responses increased for 3 subjects (1.2, 1.3 and 3.1) and was reduced or constant for the other 3 subjects (1.1, 2.1, and 2.2). Fluctuation in incorrect responding decreased for all subjects between Conditions B and C.

Four subjects (1.1, 1.2, 1.3, and 2.1) had less than 29% of overlap for correct digits between Conditions A and B. For incorrect digits, 3 subjects (1.1, 1.2, 1.3) also had low percentages of overlap (below 20%) for incorrect digits; overlap was 30% for Subject 2.1. Percentages of overlap were above the treatment effect criteria (29% for corrects and 20% for incorrects) for Subjects 2.2 and 3.1.

Percent of change in median rates (Table 21). From the beginning to the end of the investigation, all subjects increased the median level of correct digits and decreased the median level of incorrect digits. From baseline to the final condition, the percent of change in corrects ranged from 80% (Subject 3.1) to 1350% (Subject 1.3). For incorrects, the percent of change ranged from 57% (Subject 2.2) to 100% (Subject 1.1). Subjects 1.1, 1.2, and 1.3 reduced the number of incorrects by more than 90%. Subjects 2.1, 2.2, and 3.1 reduced the number of incorrects by 50% or more.

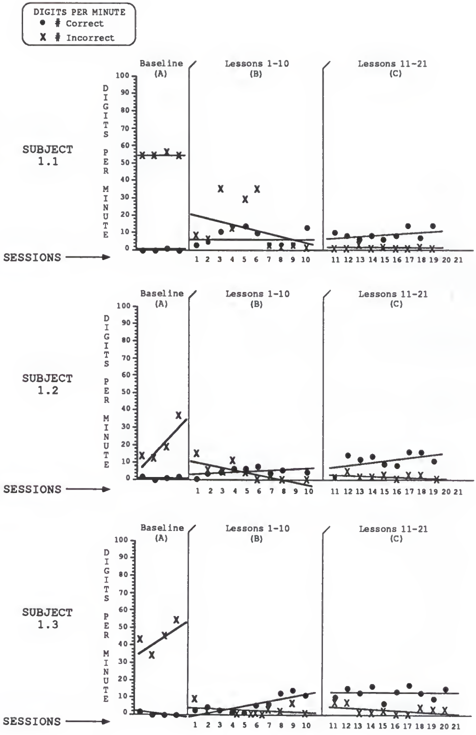


Figure 9. Trend lines for exceptional subjects in Classroom 1.

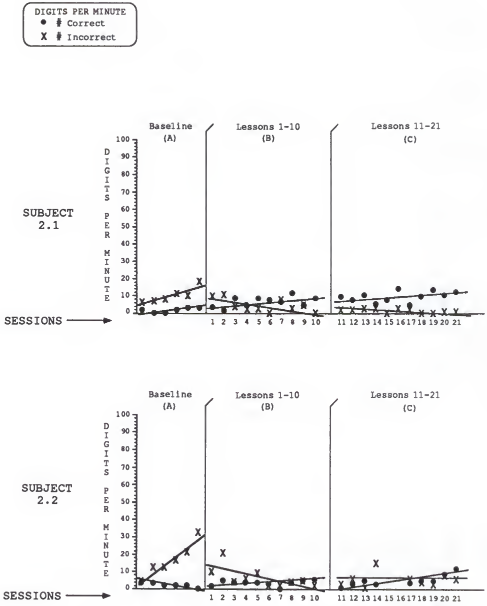


Figure 10. Trend lines for exceptional subjects in Classroom 2.

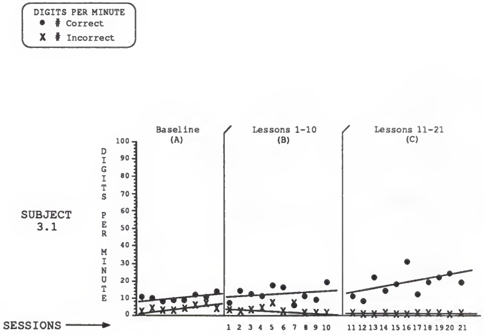


Figure 11. Trend lines for the exceptional subject in Classroom 3.

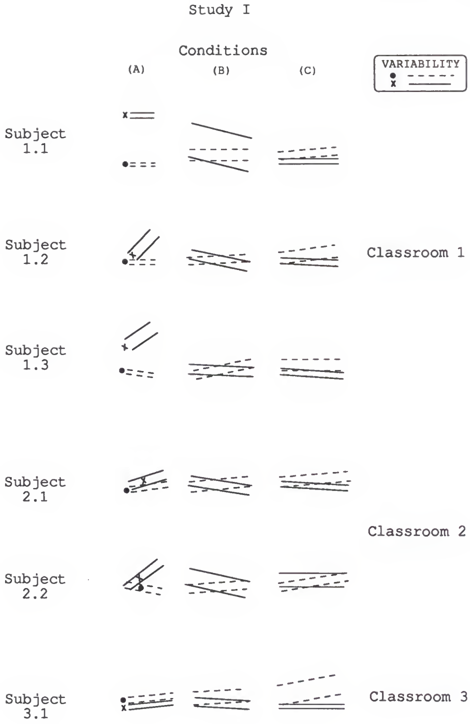


Figure 12. Variability of responding in Study I.

## Table 18

## Ranges and Median Levels for Rate in Classrooms 1, 2, and 3

| | Condition A (Baseline) | | | Condition B (Lessons 1-10) | | | Condition C (Lessons 11-21) | | |
|---|---|---|---|---|---|---|---|---|---|
| S | NDP | Range | Median | NDP | Range | Median | NDP | Range | Median |
| 1.1 | (4) | 0-1<br>54-56 | 0<br>54 | (10) | 3-14<br>1-35 | 7<br>7 | (9) | 6-14<br>0-2 | 8<br>0 |
| 1.2 | (4) | 0-2<br>12-36 | 1.5<br>15.5 | (9) | 1-8<br>0-15 | 5<br>4 | (9) | 1-16<br>0-4 | 11<br>1 |
| 1.3 | (4) | 0-2<br>34-54 | 0<br>44 | (10) | 2-15<br>0-9 | 5.5<br>1.5 | (10) | 7-18<br>0-7 | 13.5<br>2.5 |
| --- | ----- | --------- | --------- | ----- | --------- | --------- | ----- | --------- | --------- |
| 2.1 | (6) | 0-3<br>6-18 | 2<br>9 | (10) | 2-12<br>0-11 | 7.5<br>3.5 | (11) | 5-15<br>0-3 | 10<br>2 |
| 2.2 | (6) | 0-3<br>4-32 | 2<br>14 | (10) | 0-6<br>0-11 | 4<br>4.5 | (9) | 1-12<br>1-15 | 5<br>6 |
| --- | ----- | --------- | --------- | ----- | --------- | --------- | ----- | --------- | --------- |
| 3.1 | (8) | 8-14<br>2-7 | 10.5<br>4 | (10) | 6-19<br>1-7 | 11.5<br>2.5 | (11) | 8-31<br>0-1 | 19<br>1 |

S = Subject NDP = number of data points

*Note*: Broken line indicates exceptional subjects by classroom.
Correct scores are recorded over incorrect scores.

Table 19

Within Condition Summary for Question 2 in Study I

| | Condition Length (NDP) | | | Trend Corrects/ Incorrects | | | Crossover Maintained From Lesson | | | Criterion Responding Corrects/ Incorrects |
|---|---|---|---|---|---|---|---|---|---|---|
| S | (A) | (B) | (C) | (A) | (B) | (C) | (A) | (B) | (C) | |
| 1.1 | 4 | 10 | 9 | L/L | L/D | A/L | | 7 | | No/Yes |
| 1.2 | 4 | 9 | 9 | L/A | A/D | A/D | | | 12 | No/Yes |
| 1.3 | 4 | 10 | 10 | D/A | A/D | L/D | | 2 | | No/No |
| 2.1 | 6 | 10 | 11 | A/A | A/D | A/D | | 10 | | No/Yes |
| 2.2 | 6 | 10 | 9 | D/A | A/D | A/L | | | 18 | No/No |
| 3.1 | 8 | 10 | 11 | A/A | A/D | A/L | * | | | Yes/Yes |

NDP = number of data points    S = Subject
(A) = baseline  (B) = Condition B (lessons 1-10)  (C) = Condition C (lessons 11-21)
Trend: L = level  A = accelerating  D = decelerating
Note: Broken line indicates exceptional subjects by classroom.
Asterisk (*) indicates crossover on the first baseline measure.

### Table 20
### Between Condition Summary for Question 2 in Study I

| | Median Level Changes | | Changes in Trend | | Changes in Variability | | Percentage of Overlap |
|---|---|---|---|---|---|---|---|
| S | (A)-(B) | (B)-(C) | (A)-(B) | (B)-(C) | (A)-(B) | (B)-(C) | (A) and (B) |
| 1.1 | +7<br>-47 | +1<br>-7 | L-L<br>L-D | L-A<br>D-L | +<br>+ | -<br>- | 0%<br>0% |
| 1.2 | +3.5<br>-11.5 | +6<br>-3 | L-A<br>A-D | A-A<br>D-D | +<br>- | +<br>- | 11%<br>11% |
| 1.3 | +5.5<br>-42.5 | +8<br>+1 | D-A<br>A-D | A-L<br>D-D | +<br>- | +<br>- | 20%<br>0% |
| 2.1 | +5.5<br>-5.5 | +2.5<br>-1.5 | A-A<br>A-D | A-A<br>D-D | +<br>= | =<br>- | 10%<br>30% |
| 2.2 | +2<br>-9.5 | +1<br>+1.5 | D-A<br>A-D | A-A<br>D-L | +<br>+ | =<br>- | 30%<br>70% |
| 3.1 | +1<br>-1.5 | +7.5<br>-1.5 | A-A<br>A-D | A-A<br>D-L | +<br>+ | +<br>- | 50%<br>70% |

S = Subject
(A) = baseline (B) = Condition B (lessons 1-10) (C) = Condition C (lessons 11-21)
Median Level: + increased - decreased
Trend: L = level A = accelerating D = decelerating
Variability: + increased - decreased = stayed the same

*Note*: Broken line indicates exceptional subjects by classroom.
Changes in corrects are recorded over changes in incorrects.

Table 21

Changes in Median Rate Scores Across Conditions in Study I

| | Conditions | | | Conditions | | | Conditions | | | % of Change |
|---|---|---|---|---|---|---|---|---|---|---|
| S | (A) | (B) | Diff. | (B) | (C) | Diff. | (A) | (C) | Diff. | (A)-(C) |
| 1.1 | 0<br>54 | 7<br>7 | +7<br>-47 | 7<br>7 | 8<br>0 | +1<br>-7 | 0<br>54 | 8<br>0 | +8<br>-54 | +800%<br>-100% |
| 1.2 | 1.5<br>15.5 | 5<br>4 | +3.5<br>-11.5 | 5<br>4 | 11<br>1 | +6<br>-3 | 1.5<br>15.5 | 11<br>1 | +9.5<br>-14.5 | +633%<br>-94% |
| 1.3 | 0<br>44 | 5.5<br>1.5 | +5.5<br>-42.5 | 5.5<br>1.5 | 13.5<br>2.5 | +8<br>+1 | 0<br>44 | 13.5<br>2.5 | +13.5<br>-41.5 | +1350%<br>-94% |
| 2.1 | 2<br>9 | 7.5<br>3.5 | +5.5<br>-5.5 | 7.5<br>3.5 | 10<br>2 | +2.5<br>-1.5 | 2<br>9 | 10<br>2 | +8<br>-7 | +400%<br>-78% |
| 2.2 | 2<br>14 | 4<br>4.5 | +2<br>-9.5 | 4<br>4.5 | 5<br>6 | +1<br>+1.5 | 2<br>14 | 5<br>6 | +3<br>-8 | +150%<br>-57% |
| 3.1 | 10.5<br>4 | 11.5<br>2.5 | +1<br>-1.5 | 11.5<br>2.5 | 19<br>1 | +7.5<br>-1.5 | 10.5<br>4 | 19<br>1 | +8.5<br>-3 | +80%<br>-75% |

S = Subject Diff. = Difference in the median number of digits
(A) = baseline (B) = Condition B (lessons 1-10) (C) = Condition C (lessons 11-21)
% of Change: + increase - decrease

**Note**: Broken line indicates exceptional subjects by classroom.
The number of correct digits is recorded over the number of incorrect digits.

Question 3

When teachers follow the instructional procedures outlined in Multiplication Facts 0 to 81 (Mercer & Miller, 1992b) from the Strategic Math Series, will the performance of individual students with disabilities be similar to the performance of normally achieving peers in the same group on measures of accuracy and fluency?

Appendix L contains supporting documentation for the results summarized in Table 22. Pretest scores and digit writing rates on the screening instrument are provided for discussion purposes.

Classroom 1--Subject 1.1. On the pretest, Subject 1.1 scored slightly above the median score of 16 normally achieving classmates (group). Across all lessons completed during Conditions B and C (including the posttest), the accuracy performance of Subject 1.1 remained within the performance range of the group on 93% of the lessons. Scores were at or above the group median level on 33% of the lessons.

On the fluency tool skill screening, Subject 1.1 wrote fewer digits per minute than the median number written by 16 normally achieving classmates (group). Across all fluency measures of Conditions B and C, the subject's correct digit writing rate was within the group range on

84% of the measures and at or above the group median score on 31% of the measures.

Classroom 1--Subject 1.2. On the pretest, Subject 1.2 scored below the median score of 16 normally achieving classmates (group). Across all lessons completed during Conditions B and C (including the posttest), the accuracy performance of Subject 1.2 was within the group range on 100% of the lessons with performance equal to or above the group median score on 62% of the lessons.

On the fluency tool skill screening, Subject 1.2 wrote fewer digits per minute than the median number written by 16 normally achieving classmates (group). Across all fluency measures of Conditions B and C, the subject's correct digit writing rate was within the group range on 89% of the measures; 33% of the measures were at or above the group median score.

Classroom 1--Subject 1.3. On the pretest, Subject 1.3 scored below the median score of 16 normally achieving classmates (group). Across all lessons completed during Conditions B and C (including the posttest), the accuracy performance of Subject 1.3 was within the group range on 100% of the lessons with 50% of the lessons reaching or exceeding the group median score.

On the fluency tool skill screening, Subject 1.3 wrote fewer digits per minute than the median number written by 16 normally achieving classmates (group). Across all fluency measures of Conditions B and C, the subject's correct digit writing rate was within the group range on 100% of the measures; 45% of the measures were at or above the group median score.

Classroom 2--Subject 2.1. On the pretest, Subject 2.1 scored above the median score of 17 normally achieving classmates (group). Across all lessons completed during Conditions B and C (including the posttest), the accuracy performance of Subject 2.1 was within the group range on 90% of the lessons. More than half (52%) of the scores were at or above the group median score.

On the fluency tool skill screening, Subject 2.1 wrote fewer digits per minute than the median number written by 17 normally achieving classmates (group). Across all fluency measures of Conditions B and C, the subject's correct digit writing rate was within the group range on 100% of the measures; 43% of the measures were equal to or above the group median score.

Classroom 2-Subject 2.2. On the pretest, Subject 2.2 was below the median score of 17 normally achieving classmates (group). Across all lessons completed during

Conditions B and C (including the posttest), the accuracy performance of Subject 2.2 remained within the group range on 84% of the lessons with 16% of the scores equal to or above the group median score.

On the fluency tool skill screening, Subject 2.2 wrote fewer digits per minute than the median number written by 17 normally achieving classmates (group). Across all fluency measures of Conditions B and C, the subject's correct digit writing rate was within the group range on 84% of the measures. No score reached the group median score.

Classroom 3--Subject 3.1. On the pretest, Subject 3.1 scored well above the median score of 19 normally achieving classmates (group). Across all lessons completed during Conditions B and C (including the posttest), the accuracy performance of Subject 3.1 stayed within the group range on 95% of the lessons. Scores reached or exceeded the group median score on 62% of the lessons.

On the fluency tool skill screening, Subject 3.1 wrote one digit more per minute than the median number written by 19 normally achieving classmates (group). Across all fluency measures of Conditions B and C, the subject's correct digit writing rate was within or beyond the group range on 100% of the measures; 90% of the measures were at or above the group median score.

Summary of results for Question 3 (Table 22). On the pretest, 3 exceptional subjects (1.2, 1.3, and 2.2) scored below the median score of normally achieving classmates (group). Across all lessons completed during Conditions B and C (including the posttest), accuracy scores achieved by exceptional subjects were within the group range on at least 84% of the lessons. In addition, all exceptional subjects reached or exceeded the group median score on at least 16% of the lessons. Four exceptional subjects (1.2, 1.3, 2.1, and 3.1) had accuracy scores at the group median level or above on half or more of completed lessons.

During the fluency tool skill screening, 5 exceptional subjects (1.1, 1.2, 1.3, 2.1, and 2.2) wrote fewer digits per minute than the median number written by their normally achieving classmates (group). Across all fluency measures of Conditions B and C, correct digit writing rates of exceptional subjects were within the group range on at least 84% of the measures. Five exceptional subjects (1.1, 1.2, 1.3, 2.1, and 3.1) reached or exceeded the group median on at least 31% of the measures. One subject (2.2) stayed within range on 84% of the measures but failed to reach the median score on any measure.

Table 22

Individual Performance Compared to Group Performance For Question 3 in Study I

| | Accuracy | | Fluency | |
|---|---|---|---|---|
| | percentage correct scores (Conditions B and C and Posttest) | | # of correct digits on timed probes (Conditions B and C) | |
| | Percentage of Lessons | | Percentage of Measures | |
| S | within group range | group median or above | within group range | group median or above |
| 1.1 | 93% | 33% | * 84% | 31% |
| 1.2 | * 100% | 62% | * 89% | 33% |
| 1.3 | * 100% | 50% | * 100% | 45% |
| 2.1 | 90% | 52% | * 100% | 43% |
| 2.2 | * 84% | 16% | * 84% | 0% |
| 3.1 | 95% | 62% | 100% | 90% |

S = Subject

Note: Broken line indicates exceptional subjects by classroom.
Asterisk (*) indicates a score below the group median score on pretest or fluency tool skill screening.

## Study II

For Study II, results were analyzed for 7 exceptional subjects and 47 normally achieving subjects across 3 second grade classrooms. Figures and tables for the three experimental questions are grouped together following the analysis of results for each question.

### Question 1

When classroom teachers use Multiplication Facts 0 to 81 (Mercer & Miller, 1992b) from the Strategic Math Series for large-group instruction in multiplication, what are the effects on accuracy (in terms of criterion responding) for students with disabilities?

Percentage scores on each lesson are displayed in Figure 13 for subjects in Classroom 4, Figure 14 for the subject in Classroom 5, and Figure 15 for subjects in Classroom 6. Median percentage scores are presented in Figure 16. Results for Question 1 are summarized in Table 23.

Classroom 4--Subject 4.1. In Classroom 4 (Figure 13), the first classroom to begin instruction, Subject 4.1 scored 15% on the pretest. The entire C-R-A sequence was completed during Condition B, and the first and third lessons were repeated. On lesson 1, the first score was 40%. After reteaching, the subject scored 100%. The first score on lesson 3 was 60%; on the second attempt, the subject reached 100%. The posttest score was 100%, an increase of 85 percentage points from the pretest. The subject did not complete lessons 15 and 18 of Condition C. On completed lessons, scores ranged from 70% to 100%. Median performance levels were 90% during Condition B and 80% during Condition C (see Figure 16).

Classroom 4--Subject 4.2. Subject 4.2 scored 20% on the pretest. The subject completed the entire C-R-A sequence during Condition B and exceeded the 80% criterion score on all lessons. On the posttest, the subject scored 100%, an increase of 80 percentage points from the pretest. The subject did not complete lessons 18 and 20 of Condition C. On completed lessons, scores ranged from 70% to 100%. Median performance levels were 90% during Condition B and 100% during Condition C (see Figure 16).

Classroom 4--Subject 4.3. Subject 4.3 obtained a score of 15% on the pretest. The subject completed the entire C-R-A sequence during Condition B. Lesson 1 was the only repeated lesson. The subject's first score on lesson 1 was 70%; on the second attempt, the subject reached 90%. The subject scored 95% on the posttest, an increase of 80 percentage points from the pretest. The subject did not complete lessons 17 and 18 of Condition C. On completed lessons, scores ranged from 40% to 100%. Median performance levels were 90% for Condition B and 80% for Condition C (see Figure 16).

Classroom 5--Subject 5.1. In Classroom 5 (Figure 14), the second classroom to begin instruction, Subject 5.1 scored 5% on the pretest. The subject completed the entire C-R-A sequence during Condition B. Scores on lessons 6 and 8 were below the 80% criterion but, due to absences, the lessons were not repeated. The subject required three attempts to reach the posttest criterion. The first two scores were 60% and 75%. On the third attempt, the subject reached 90%. The first posttest score represented an increase of 55 percentage points from the pretest. With more consistent attendance during Condition C, the subject completed all lessons except lesson 14. Scores ranged from 30% to 100%. Median performance levels were 90% for Condition B and 60% for Condition C (see Figure 16).

Classroom 6--Subject 6.1. In Classroom 6 (Figure 15), the third classroom to begin instruction, Subject 6.1 scored 20% on the pretest. The subject completed the entire C-R-A sequence during Condition B and exceeded the 80% criterion score on all lessons except lesson 8. On lesson 8, the subject scored 70%. Due to absence, the lesson was not repeated. The subject's first posttest score was 70%, an increase of 50 percentage points from the pretest score. Following feedback and reteaching of lesson 10, the subject's score was 100%. The subject completed all lessons except lesson 21 of Condition C. On completed lessons, scores ranged from 80% to 100%. Median performance levels were 100% for Condition B and 90% for Condition C (see Figure 16).

Classroom 6--Subject 6.2. Subject 6.2 had no correct answers (0%) on the pretest. The subject completed the C-R-A sequence except for lesson 8 during Condition B. Only lesson 1 was repeated. The first score on lesson 1 was 10%. Following reteaching, the score was 100%. The subject's first score on the posttest was 70%, an increase of 70 percentage points from the pretest score. On the second attempt, the subject scored 100%. During Condition C, lessons 12 and 21 were not completed. On completed lessons, scores ranged from 50% to 90%. Median performance levels were 95% for Condition B and 80% for Condition C (see Figure 16).

Classroom 6--Subject 6.3. Subject 6.3 scored 5% on the pretest. The subject completed the entire C-R-A sequence during Condition B. Lesson 1 was repeated. The subject's first score on lesson 1 was 50%. Following reteaching, the subject scored 100%. The subject's first score on the posttest was 65%, an increase of 60 percentage points from the pretest. Following feedback and reteaching of lesson 10, the subject scored 100%. During Condition C, several lessons (11, 14, 15, 16, 17, and 19) were not completed. On the completed lessons, scores ranged from 70% to 100%. Median performance levels were 90% for Condition B and 80% for Condition C (see Figure 16).

Summary of results for Question 1. The results for Question 1 are summarized in Table 23. Pretest scores ranged from 0% to 20% and indicated the need for instruction in multiplication. During Condition B, 6 subjects completed the entire C-R-A sequence. Subject 6.2 completed eight of the nine lessons. No subject scored below the criterion on more than two lessons in Condition B. Two subjects (5.1 and 6.1) were allowed to continue instruction without repeating lessons that were below the criterion. Subjects reached the 80% criterion on the first attempt on at least 78% of the lessons, and median scores ranged from 90% to 100%.

All subjects improved from pretest to posttest. The amount of improvement ranged from 50 to 85 percentage points. Scores for subjects' first attempts on the posttest ranged from 60% to 100%. Three subjects (4.1, 4.2, and 4.3) reached the posttest criterion on the first attempt. Following feedback and reteaching of lesson 10, 3 other subjects (6.1, 6.2, and 6.3) reached the posttest criterion. One subject (5.1) required an additional reteaching session to reach the criterion.

With the introduction of complex word problems in Condition C, scores were generally lower than in the previous condition. Six subjects (4.1, 4.2, 4.3, 5.1, 6.2, and 6.3) achieved criterion responding for at least 40% of the lessons in Condition C. Subject 6.1 reached the criterion score on 100% of lessons completed during Condition C. For 6 subjects (4.1, 4.3, 5.1, 6.1, 6.2, and 6.3), median scores ranged from 60% to 90%. The median score for Subject 4.2 was 100%--higher than the subject's median score during the first condition.

### Question 2

When classroom teachers use Multiplication Facts 0 to 81 (Mercer & Miller, 1992b) from the Strategic Math Series for large-group instruction in multiplication, what are the effects on fluency (in terms of criterion responding) for students with disabilities?

Individual results are displayed by classroom (refer again to Figures 13, 14, and 15). Computer-generated trend lines are presented by classroom in Figures 17, 18, and 19.

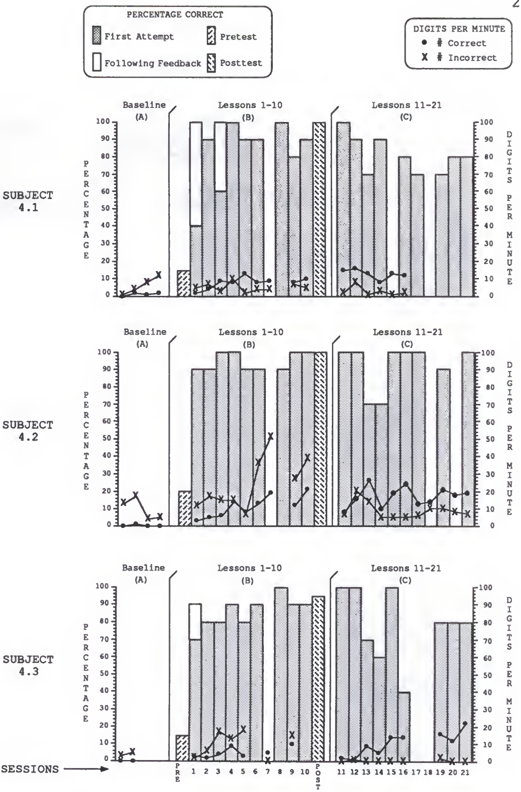


Figure 13. Percentage and rate data for exceptional subjects in Classroom 4.

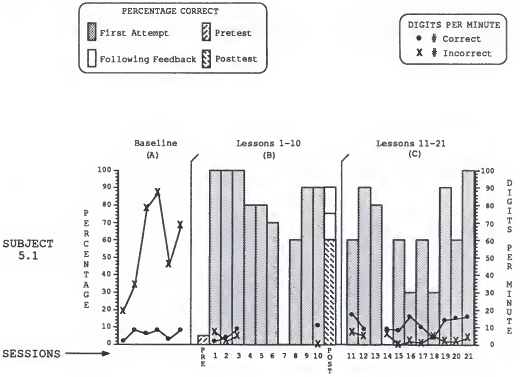


Figure 14. Percentage and rate data for the exceptional subject in Classroom 5.

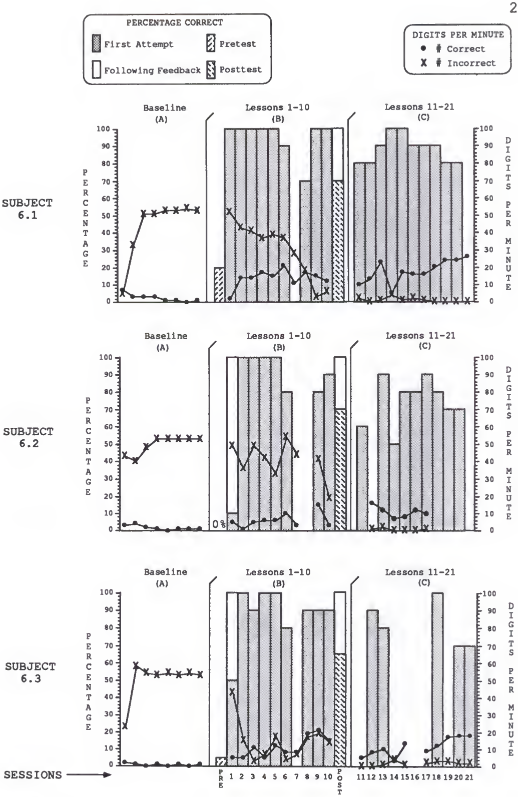


Figure 15. Percentage and rate data for exceptional subjects in Classroom 6.

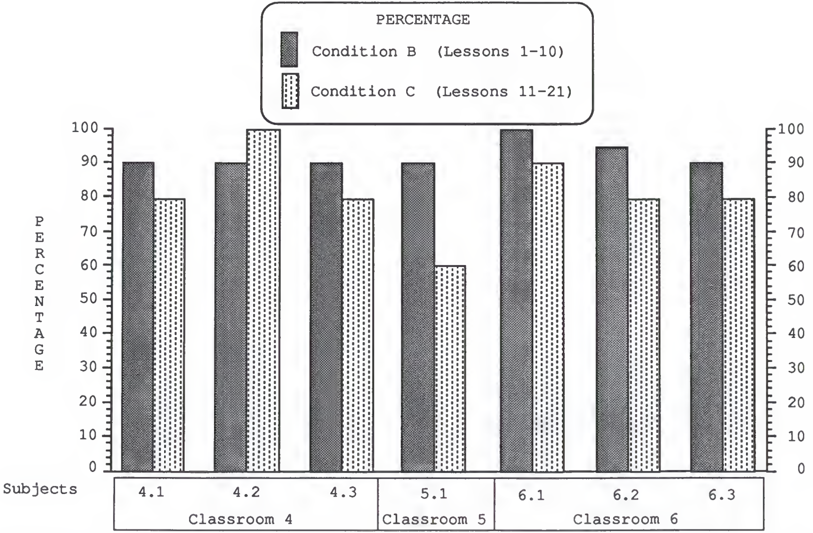


Figure 16. Median percentage scores for exceptional subjects in Classrooms 4, 5, and 6.

Table 23

Summary of Results for Question 1 in Study II

| S | Pretest | Posttest | Increase Pre-Post* | # Lessons Repeated (B) | Lessons at Criterion (B)* | Median % (B) | Lessons at Criterion (C) | Median % (C) |
|---|---|---|---|---|---|---|---|---|
| 4.1 | 15% | 100% | 85% | 2 of 9 | 78% | 90% | 67% | 80% |
| 4.2 | 20% | 100% | 80% | 0 of 9 | 100% | 90% | 78% | 100% |
| 4.3 | 15% | 95% | 80% | 1 of 9 | 89% | 90% | 67% | 80% |
| 5.1 | 5% | 60% 75% 90% | 55% | 2 of 9** | 78% | 90% | 40% | 60% |
| 6.1 | 20% | 70% 100% | 50% | 1 of 9** | 89% | 100% | 100% | 90% |
| 6.2 | 0% | 70% 100% | 70% | 1 of 8 | 87% | 95% | 55% | 80% |
| 6.3 | 5% | 65% 100% | 60% | 1 of 9 | 89% | 90% | 60% | 80% |

S = Subject * = first attempt scores only

(B) = Condition B (lessons 1-10) (C) = Condition C (lessons 11-21)

Note: Broken line indicates exceptional subjects by classroom.
Posttest column shows all attempts to reach the 90% criterion score.
Two asterisks (**) indicate lessons below criterion that were not repeated.

Frequency envelopes for visual analysis of variability are shown in Figure 20. Ranges and median performance levels are provided in Table 24. A summary of within condition analyses is provided in Table 25. Between condition analyses are summarized in Table 26. Table 27 contains the overall percent of change in median rate scores.

After four consecutive baseline measures, it was evident that performance was unchanged or deteriorating for 6 of the 7 exceptional subjects across three classrooms. Even the subject in Classroom 4 (Subject 4.2) who reduced the number of errors made no improvement in the number of correct digits. Consequently, the intervention was introduced into Classroom 4. One subject in Classroom 4 was absent during the last 2 days of baseline but returned in time to begin instruction. Two days were required for exceptional subjects to reach criterion responding on the first two lessons in Classroom 4. During those 2 days, Classrooms 5 and 6 remained under baseline conditions. The intervention was then introduced into Classroom 5 while subjects in Classroom 6 continued baseline monitoring. After the exceptional subjects in Classroom 5 reached criterion responding on the first two lessons (2 days), the intervention was introduced into Classroom 6. During the extended baselines in Classrooms 5 and 6, performance did not improve for any subject.

Classroom 4--Subject 4.1. Within condition analysis for Subject 4.1 included four measures during Condition A, nine measures during Condition B, and six measures during Condition C. With correct digits recorded over incorrect digits, median performance levels were 1.5/6 during Condition A, 8/5 in Condition B, and 13/2 for Condition C (Table 24).

Trend lines for each condition are shown in Figure 17. During Condition A, correct responses accelerated slightly as incorrect responses accelerated more rapidly. During Condition B, there was improvement for both corrects (slight acceleration) and incorrects (slight deceleration). During Condition C, decelerating trends were noted for both correct and incorrect responding.

Crossover was maintained following lesson 5. With a high score of 16 correct digits on lesson 12, the subject failed to meet the criterion of 30 for correct responding. With incorrect responses at two or below on the final two measures, the subject did meet the criterion for incorrect digits.

Examination of Table 24 indicates that Subject 4.1 increased the median level of corrects by 6.5 digits from

Condition A to Condition B and by 5 digits between Conditions B and C. Between Conditions A and B, the median level for incorrects decreased by 1 digit. An additional decrease of 3 digits took place between Conditions B and C.

Changes in trend are shown for Subject 4.1 in Figure 17. Correct responding accelerated slightly during Condition A, accelerated more rapidly during Condition B, and decelerated during condition C. Incorrect responding, accelerating steadily during Condition A, took a downward turn during Condition B, and continued to decelerate during Condition C.

Visual inspection of frequency envelopes than enclose all of the data points in each condition (Figure 20) indicates that variability increased for both correct and incorrect responses between Conditions A and B. Between Conditions B and C, variability decreased for correct responses and remained the same for incorrect responses. Between Conditions A and B, 11% of the correct responses and 100% of the incorrect responses overlapped the two conditions.

Classroom 4--Subject 4.2. Within condition analysis for Subject 4.2 included 4 measures during Condition A, 9 measures during Condition B, and 11 measures during Condition C. With correct digits recorded over incorrect digits, median performance levels were 0/9 during Condition A, 12/17 during Condition B, and 18/7 in Condition C (Table 24).

Trend lines for within condition analysis are shown in Figure 17. During Condition A, correct responding remained level while incorrect responding decelerated. During Condition B, both correct and incorrect responding accelerated rapidly. During Condition C, correct responding continued to accelerate slightly as incorrect responding reversed and began to decelerate.

Crossover was maintained following lesson 13. The subject's highest correct score of 26 digits occurred at lesson 13, so the subject did not meet the criterion of 30 for correct digits. With the number of incorrect digits never falling lower than five, the subject also failed to meet the criterion for incorrect digits.

Inspection of Table 24 shows that Subject 4.2 increased the median number of corrects by 12 digits between Conditions A and B. An additional increase of 6 digits occurred between Conditions B and C. The median number of incorrects increased by 8 digits between Conditions A and B and decreased by 10 digits between Conditions B and C.

Changes in trend between conditions are shown for Subject 4.2 in Figure 17. Correct responding, stable at a very low level during Condition A, accelerated during Condition B. During Condition C, correct responding dropped to a lower level than at the end of Condition B but continued a slight upward trend. Incorrect responding decelerated during Condition A and then accelerated rapidly during Condition B. The upward trend was reversed during Condition C.

Visual inspection of frequency envelopes that encompass all of the data points in each condition (Figure 20) confirms that variability increased for both correct and incorrect responses between Conditions A and B. Between Conditions B and C, variability increased for correct responses and decreased for incorrect responses. There were no overlapping correct responses between Conditions A and B. Incorrect responses overlapped the two conditions by 55%.

Classroom 4--Subject 4.3. Due to absence at the beginning of the investigation, within condition analysis for Subject 4.3 included only two measures during Condition A, seven measures during Condition B, and nine measures in Condition C. With correct digits recorded over incorrect digits, median performance levels were 0/4 during Condition

A, 4/13 in Condition B, and 12/0 during Condition C (Table 24).

Trend lines for each condition are shown in Figure 17. During Condition A, there were not enough data points to establish performance trends. During Condition B, both correct and incorrect responses were accelerating. During Condition C, correct responses accelerated as incorrect responses stabilized at a low level.

Crossover was maintained following lesson 13. The subject's high score of 22 correct digits occurred on lesson 21 and did not meet the criterion of 30 for correct responding. With 2 or fewer incorrect responses from lesson 11 on, the subject met the criterion for incorrect digits.

Examination of Table 24 shows that Subject 4.3 increased the number of correct responses by 4 digits between Conditions A and B and by 8 digits between Conditions B and C. Incorrect responses increased by 9 digits between Conditions A and B and decreased by 13 digits between Conditions B and C.

Changes in trend between conditions are shown for Subject 4.3 in Figure 17. With only two data points in Condition A, no trend was established. During Condition B, correct responding showed an accelerating trend that dropped to a lower accelerating level during Condition C. Incorrect responding showed an accelerating trend during Condition B that stopped and stabilized at a low level during Condition C.

Visual inspection of frequency envelopes than enclose all of the data points in each condition (Figure 20) indicates that variability of responding increased for corrects and decreased for incorrects between Conditions B and C. With only two measures in Condition A, percentages of overlap could not be calculated.

Classroom 5--Subject 5.1. Within condition analysis for Subject 5.1 included 6 measures in Condition A, 4 measures for Condition B, and 10 measures during Condition C. With correct digits recorded over incorrect digits, median performance levels were 7/57 during Condition A, 6.5/3.5 in Condition B, and 12/3 during Condition C (Table 24).

Trend lines for each condition are shown in Figure 18. During Condition A, both correct and incorrect responses were accelerating. Acceleration was more rapid for incorrects. During Condition B, correct responses accelerated as incorrect responses decelerated. An accelerating trend for correct responses with a decelerating trend for incorrect responses was also the pattern during Condition C.

Crossover was demonstrated several times, but responding was inconsistent through both instructional conditions. Crossover was not maintained until lesson 19. The subject's highest number of correct digits was 17 on lesson 11, so the subject did not meet the criterion of 30 for correct digits. With more than two errors all the way through Condition C, the subject also failed to meet the criterion for incorrect digits.

Examination of Table 24 shows that Subject 5.1 decreased the median number of correct responses by .5 digits between Conditions A and B. An increase of 5.5 digits took place between Conditions B and C. Incorrect responses decreased by 53.5 digits between Conditions A and B and decreased by another .5 digits between Conditions B and C.

Changes in trend between conditions are displayed for Subject 5.1 in Figure 18. Correct responding accelerated slightly during Condition A and continued an upward trend during Condition B. During Condition C, the trend continued to accelerate but at a lower level and less steeply than in the previous condition. Incorrect responding, accelerating rapidly during Condition A, dropped to a lower level and decelerated during Condition B. The downward trend continued during Condition C but at a higher level and less steeply than in Condition B.

Visual inspection of frequency envelopes that encompass all of the data points in each condition (Figure 20) indicates that variability decreased for both correct and incorrect responding between Conditions A and B. Between Conditions B and C, variability increased for correct responses and remained about the same for incorrect responses. Between Conditions A and B, half (50%) of the correct and none (0%) of the incorrect data points overlapped the two conditions.

Classroom 6--Subject 6.1. Within condition analysis for Subject 6.1 focused on 8 measures during Condition A, 10 measures during Condition B, and 11 measures during Condition C. With correct responses recorded over incorrect responses, median performance levels were 2/52 for Condition A, 14.5/37 in Condition B, and 17/1 during Condition C (Table 24).

Trend lines for each condition are shown in Figure 19. In Condition A, correct responses decelerated as incorrect responses accelerated. The trends were reversed during Condition B. During Condition C, correct responses continued to accelerate as incorrect responses decelerated.

Crossover was maintained following lesson 9. The subject's highest correct score was 26 digits on lesson 21, so the subject did not meet the criterion of 30 for correct digits. With two or fewer incorrect responses from lesson 15 on, the subject did reach the criterion for incorrect digits.

Inspection of Table 24 indicates that Subject 6.1 increased the median number of corrects by 12.5 digits between Conditions A and B and by another 2.5 digits between Conditions B and C. Incorrects were reduced by 15 digits between Conditions A and B and by an additional 36 digits between Conditions B and C.

Changes in trend are shown for Subject 6.1 in Figure 19. Correct responding had a decelerating trend during Condition A that was reversed during Condition B. Correct responses began at a slightly lower level during Condition C but showed a steeper upward trend than in the previous condition. The rapidly accelerating trend for incorrect responses during Condition A was reversed in Condition B. A slight downward trend continued for incorrect responses during Condition C.

Visual inspection of frequency envelopes that enclose all of the data points in each condition (Figure 20) indicates increased variability for correct responses and decreased variability for incorrect responses between Conditions A and B. Between Conditions B and C, variability remained about the same for correct responses and decreased for incorrect responses. Percentages of overlap were low for correct responses (10%) and high for incorrect responses (90%) between Conditions A and B.

Classroom 6--Subject 6.2. Within condition analysis for Subject 6.2 included eight measures during Condition A, nine measures for Condition B, and six measures during Condition C. With correct digits recorded over incorrect digits, median performance levels were 1/53 in Condition A, 5/42 during Condition B, and 11/.5 during Condition C (Table 24).

Trend lines for each condition are shown in Figure 19. During Condition A, correct responses decelerated as the trend for incorrect responses accelerated. The trends were reversed during Condition B. In Condition C, correct responses began at a slightly higher level but decelerated. Incorrect responses dropped to a much lower level and continued to decelerate.

Crossover was maintained following lesson 12. With a high score of 16 correct digits (lesson 12), the subject did not meet the criterion of 30 for correct responding. With the number of incorrect responses reduced during

Condition C, the subject did meet the criterion for incorrect responding.

Examination of Table 24 indicates that Subject 6.2 increased the median level of correct responses by 4 digits between Conditions A and B and by 6 digits between Conditions B and C. The median level of incorrect responses was reduced by 11 digits between Conditions A and B and by an additional 41.5 digits between Conditions B and C.

Changes in trend between conditions are shown for Subject 6.2 in Figure 19. Correct responding decelerated during Condition A, accelerated during Condition B, and reverted to a downward trend in Condition C. Incorrect responding accelerated during Condition A, decelerated during Condition B, and leveled off after slight deceleration in Condition C.

Visual inspection of frequency envelopes that encompass all of the data points in each condition (Figure 20) indicates that variability increased for both correct and incorrect responses between Conditions A and B. Between Conditions B and C, variability decreased for both correct responses and incorrect responses. The percentage of overlapping data points between Conditions A and B was

33% for correct responses. Incorrect responses overlapped the two conditions by 55%.

Classroom 6--Subject 6.3. Within condition analysis for Subject 6.3 included 8 measures in Condition A, 10 measures during Condition B, and 10 measures for Condition C. With correct digits recorded over incorrect digits, median performance levels were 1/53.5 in Condition A, 9.5/14.5 during Condition B, and 11/2 in Condition C (Table 24).

Trend lines for each condition are shown for Subject 6.3 in Figure 19. In Condition A, the trend for correct responses decelerated slightly as the trend for incorrect responses accelerated. The trends were reversed during Condition B. During Condition C, correct responses dropped to a lower level but continued to accelerate. Incorrect responses also dropped to a lower level but showed a slightly accelerating trend.

Crossover was maintained following lesson 15. With a high score of 16 correct digits (lesson 12), the subject did not meet the criterion of 30 for correct responding. With two or fewer errors on 7 of the 10 lessons completed Condition C, the subject did meet the criterion for incorrect responding.

Inspection of Table 24 shows that Subject 1.3 increased the median number of corrects by 8.5 digits between Conditions A and B and by an additional 1.5 digits between Conditions B and C. The median number of incorrect responses was reduced by 39 digits between Conditions A and B and by 12.5 digits between Conditions B and C.

Changes in trend between conditions are shown for Subject 6.3 in Figure 19. Correct responding, almost level with a slight deceleration during Condition A, accelerated during Condition B, and dropped but continued to accelerate during Condition C. Incorrect responding accelerated during Condition A, decelerated during Condition B, and dropped to a lower level with a slightly accelerating trend during Condition C.

Visual inspection of frequency envelopes that enclose all of the data points in each condition (Figure 20) indicates that variability increased for correct responses and remained about the same for incorrect responses between Conditions A and B. Between Conditions B and C, variability remained about the same for correct responses and decreased for incorrect responses. The percentage of overlapping data points between Conditions A and B was 0% for correct responses and 10% for incorrect responses.

Within condition summary for Question 2 (Table 25). Baseline conditions included four measures for 2 subjects in Classroom 4, two measures for 1 subject in Classroom 4, six measures in Classroom 5, and eight measures in Classroom 6. During Condition B, 6 (4.1, 4.2, 4.3, 6.1, 6.2, and 6.3) of the 7 subjects had at least seven rate measures. One subject (5.1) had four rate measures. During Condition C, 5 subjects (4.2, 4.3, 5.1, 6.1, and 6.3) had nine or more measures. Two subjects (4.1 and 6.2) each had six rate measures.

All subjects began the investigation (Condition A) with the median number of correct digits lower than the median number of incorrect digits (refer again to Table 24). During Condition B, only 2 subjects (4.1 and 5.1) increased the median number of corrects above the median number of incorrects. During Condition C, the other 5 subjects (4.2, 4.3, 6.1, 6.2, and 6.3) increased the level of corrects above the level of incorrects. All subjects ended the investigation with the median for correct digits higher than the median for incorrect digits.

Analysis of trend lines within conditions indicates that correct responding was either not improving or improving very slowly as incorrect responding accelerated rapidly for 5 subjects (4.1, 5.1, 6.1, 6.2, and 6.3) during

Condition A. Subject 4.2 showed a decelerating trend for incorrect responses but no change for correct responses. Subject 4.3 had too few responses to establish a trend. For all subjects, incorrect responses remained above correct responses during Condition A. During Condition B, correct responses accelerated and incorrect responses decelerated for 5 subjects (4.1, 5.1, 6.1, 6.2, and 6.3). Two subjects (4.2 and 4.3) had accelerating correct and incorrect responses. In Condition C, 5 subjects (4.2, 4.3, 5.1, 6.1, and 6.3) had accelerating correct and low or decelerating incorrect responses. Two subjects (4.1 and 6.2) ended with decelerating trends for both correct and incorrect responses.

Crossover was demonstrated at different points during instruction. The effect was observed for 2 subjects during Condition B: Subject 4.1 (lesson 5--representational phase) and Subject 6.1 (lesson 9--abstract phase). Three subjects reached crossover early in Condition C: Subject 6.2 (lesson 12) and Subjects 4.2 and 4.3 at lesson 13. Subject 6.3 reached crossover following lesson 15. Subject 5.1 demonstrated crossover at several points, but the effect was not maintained until lesson 19.

No subject reached the criterion of 30 for correct digits by the end of the investigation. However, 1 subject

(6.1) reached 26 correct digits per minute with incorrect digits reduced to zero. Five subjects (4.1, 4.3, 6.1, 6.2, and 6.3) reached the two or fewer criterion for incorrect digits. The two subjects (4.2 and 5.1) who did not reach the incorrect digit criterion both had decelerating incorrects at the end of the investigation.

Between condition summary for Question 2 (Table 26). Examination of changes in median scores across adjacent conditions confirms that 4 subjects (4.1, 6.1, 6.2, and 6.3) increased the number of correct digits and reduced the number of incorrect digits between Conditions A and B. Subjects 4.2 and 4.3 increased both correct and incorrect digits, and 1 subject (5.1) had a slight decrease in correct digits with a substantial reduction in the number of incorrect digits. All subjects increased the number of correct digits and reduced the number of incorrect digits between Conditions B and C.

In general, trends showed correct responses that were not improving and accelerating incorrect responses during Condition A (Subjects 4.1, 5.1, 6.1, 6.2, and 6.3). Only 1 subject (4.2) had decelerating incorrects; however, corrects were very low (1 or below) and showed no change. When the intervention was introduced into each classroom (Condition B), all subjects showed accelerating trends for correct responding. Incorrect responses decelerated simultaneously for 5 subjects (4.1, 5.1, 6.1, 6.2, and 6.3). Subject 4.2 had accelerating incorrects during Condition B. Trend direction could not be established for Subject 4.3, so effects could not be analyzed between Conditions A and B. When fluency practice and feedback on the probes became part of instruction (Condition C), 4 subjects (4.2, 4.3, 5.1, and 6.1) had accelerating correct responses with decelerating or level (below the criterion) incorrect responses. Subject 6.3 had a slightly accelerating trend for incorrects. Subjects 4.1 and 6.2 had decelerating trends for both correct and incorrect responses.

Visual inspection of performance fluctuation around trend lines indicates that 3 (4.1, 4.2, and 6.3) of the 6 subjects with sufficient measures for baseline trends increased the amount of variability in both correct and incorrect responding between Conditions A and B. Two subjects (6.1 and 6.3) had increased variability for correct responses and reduced variability for incorrect responses. Another subject (5.1) had reduced variability for both correct and incorrect responses. Between Conditions B and C, variability of correct responding increased for 3 subjects (4.2, 4.3, and 5.1) and decreased or remained the same for 4 subjects (4.1, 6.1, 6.2, and 6.3). Variability of incorrect responding remained about the same or decreased for all 7 subjects.

Percentages of overlap between Conditions A and B were calculated for 6 subjects. Percentages were low (less than 29%) for 4 (4.1, 4.2, 6.1, and 6.3) subjects on correct responding. Subject 5.1 had 50% overlap for correct responses, and Subject 6.2 had 33% of overlap. For incorrect responding, percentages of overlap were below 20% for 2 subjects (5.1 and 6.3). Percentages of overlap for incorrect responses were high for Subjects 4.1 (100%), 4.2 (55%), 6.1 (90%), and 6.2 (55%).

Percent of change in median scores (Table 27). From the beginning to the end of the investigation, all subjects increased the median level of correct digits and decreased the median level of incorrect digits. From baseline to the final condition, the percent of change for correct digits ranged from 71% (Subject 5.1) to 1800% (Subject 4.2). For incorrect digits, the percent of change ranged from 22% (Subject 4.2) to 100% (Subject 4.3). Five subjects (4.3, 5.1, 6.1, 6.2, and 6.3) decreased the number of incorrect responses by at least 95%. Subject 4.1 reduced the number of incorrect responses by 66%. Subject 4.2 only showed a 22% reduction in the number of incorrect digits.

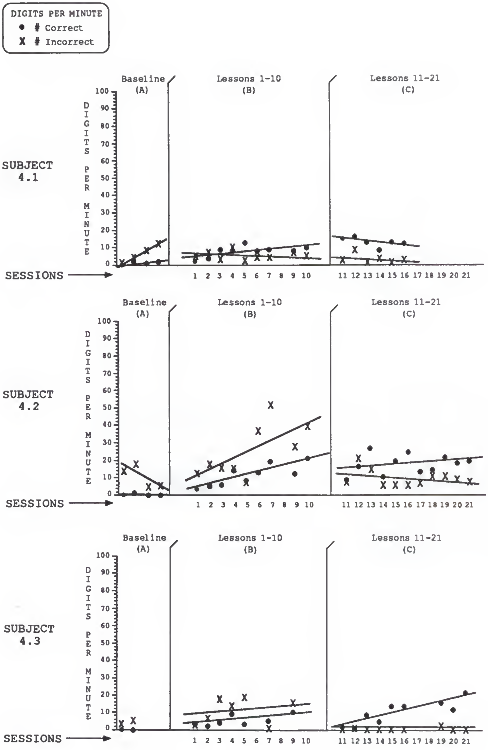


Figure 17. Trend lines for exceptional subjects in Classroom 4.

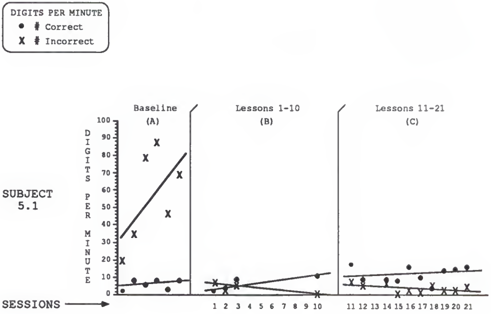


Figure 18. Trend lines for the exceptional subject in Classroom 5.

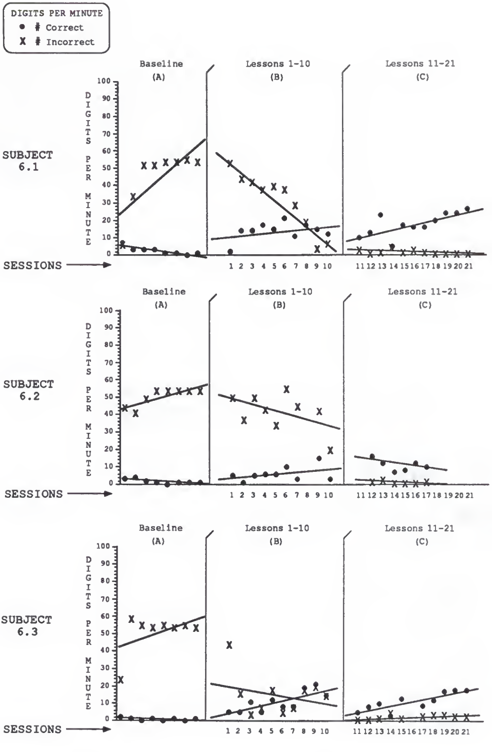


Figure 19. Trend lines for exceptional subjects in Classroom 6.

Study II

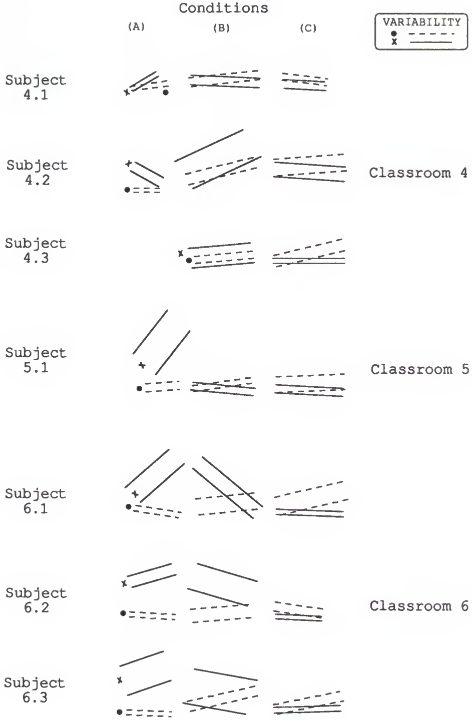


Figure 20. Variability of responding in Study II.

Table 24

Ranges and Median Levels for Rate in Classrooms 4, 5, and 6

| | Condition A (Baseline) | | | Condition B (Lessons 1-10) | | | Condition C (Lessons 11-21) | | |
|---|---|---|---|---|---|---|---|---|---|
| S | NDP | Range | Median | NDP | Range | Median | NDP | Range | Median |
| 4.1 | (4) | 0-2<br>1-12 | 1.5<br>6 | (9) | 2-13<br>2-10 | 8<br>5 | (6) | 8-16<br>1-8 | 13<br>2 |
| 4.2 | (4) | 0-1<br>4-17 | 0<br>9 | (9) | 3-21<br>7-51 | 12<br>17 | (11) | 8-26<br>5-20 | 18<br>7 |
| 4.3 | (2) | 0;0<br>3;5 | 0<br>4 | (7) | 2-10<br>0-18 | 4<br>13 | (9) | 1-22<br>0-2 | 12<br>0 |
| 5.1 | (6) | 2-8<br>19-87 | 7<br>57 | (4) | 2-11<br>0-7 | 6.5<br>3.5 | (10) | 4-17<br>0-7 | 12<br>3 |
| 6.1 | (8) | 0-7<br>5-54 | 2<br>52 | (10) | 2-21<br>3-52 | 14.5<br>37 | (11) | 5-26<br>0-4 | 17<br>1 |
| 6.2 | (8) | 0-4<br>40-53 | 1<br>53 | (9) | 1-15<br>19-54 | 5<br>42 | (6) | 7-16<br>0-2 | 11<br>.5 |
| 6.3 | (8) | 0-2<br>23-58 | 1<br>53.5 | (10) | 5-21<br>3-43 | 9.5<br>14.5 | (10) | 3-18<br>0-4 | 11<br>2 |

S = Subject     NDP = number of data points

Note: Broken line indicates exceptional subjects by classroom.
Correct scores are recorded over incorrect scores.

Table 25
Within Condition Summary for Question 2 in Study II

| S | Condition Length (NDP) |  |  | Trend Corrects/ Incorrects |  |  | Crossover Maintained From Lesson |  |  | Criterion Responding Corrects/ Incorrects |
|---|---|---|---|---|---|---|---|---|---|---|
|  | (A) | (B) | (C) | (A) | (B) | (C) | (A) | (B) | (C) |  |
| 4.1 | 4 | 9 | 6 | A/A | A/D | D/D |  | 5 |  | No/Yes |
| 4.2 | 4 | 9 | 11 | L/D | A/A | A/D |  |  | 13 | No/No |
| 4.3 | 2 | 7 | 9 | NED | A/A | A/L |  |  | 13 | No/Yes |
| 5.1 | 6 | 4 | 10 | A/A | A/D | A/D |  |  | 19 | No/No |
| 6.1 | 8 | 10 | 11 | D/A | A/D | A/D |  | 9 |  | No/Yes |
| 6.2 | 8 | 9 | 6 | D/A | A/D | D/D |  |  | 12 | No/Yes |
| 6.3 | 8 | 10 | 10 | D/A | A/D | A/A |  |  | 15 | No/Yes |

NDP = number of data points NED = not enough data points S = Subject
(A) = baseline (B) = Condition B (lessons 1-10) (C) = Condition C (lessons 11-21)
Trend: L = level A = accelerating D = decelerating
Note: Broken line indicates exceptional subjects by classroom.

## Table 26

## Between Condition Summary for Question 2 in Study II

| | Median Level Changes | | Changes in Trend | | Changes in Variability | | Percentage of Overlap |
|---|---|---|---|---|---|---|---|
| S | (A)-(B) | (B)-(C) | (A)-(B) | (B)-(C) | (A)-(B) | (B)-(C) | (A) and (B) |
| 4.1 | +6.5 | +5 | A-A | A-D | + | - | 11% |
| | -1 | -3 | A-D | D-D | + | = | 100% |
| 4.2 | +12 | +6 | L-A | A-A | + | + | 0% |
| | +8 | -10 | D-A | A-D | + | - | 55% |
| 4.3 | +4 | +8 | NED | A-A | NED | + | NED |
| | +9 | -13 | | A-L | | - | |
| 5.1 | -.5 | +5.5 | A-A | A-A | - | + | 50% |
| | -53.5 | -.5 | A-D | D-D | - | = | 0% |

Table 26--Continued

|  | Median Level Changes |  | Changes in Trend |  | Changes in Variability |  | Percentage of Overlap |
|---|---|---|---|---|---|---|---|
| S | (A)-(B) | (B)-(C) | (A)-(B) | (B)-(C) | (A)-(B) | (B)-(C) | (A) and (B) |
| 6.1 | +12.5 | +2.5 | D-A | A-A | + | = | 10% |
|  | -15 | -36 | A-D | D-D | - | - | 90% |
| 6.2 | +4 | +6 | D-A | A-D | + | - | 33% |
|  | -11 | -41.5 | A-D | D-D | + | - | 55% |
| 6.3 | +8.5 | +1.5 | D-A | A-A | + | = | 0% |
|  | -39 | -12.5 | A-D | D-A | = | - | 10% |

S = Subject NED = not enough data points
(A) = baseline (B) = Condition B (lessons 1-10) (C) = Condition C (lessons 11-21)
Median Level: + increased - decreased
Trend: L = level A = accelerating D = decelerating
Variability: + increased - decreased = stayed the same
Note: Broken line indicates exceptional subjects by classroom.
Changes in corrects are recorded over changes in incorrects.

Table 27

Changes in Median Rate Scores Across Conditions in Study II

| | Conditions | | | Conditions | | | Conditions | | | % of Change |
|---|---|---|---|---|---|---|---|---|---|---|
| S | (A) | (B) | Diff. | (B) | (C) | Diff. | (A) | (C) | Diff. | (A)-(C) |
| 4.1 | 1.5 | 8 | +6.5 | 8 | 13 | +5 | 1.5 | 13 | +11.5 | +766% |
| | 6 | 5 | -1 | 5 | 2 | -3 | 6 | 2 | -4 | -66% |
| 4.2 | 0 | 12 | +12 | 12 | 18 | +6 | 0 | 18 | +18 | +1800% |
| | 9 | 17 | +8 | 17 | 7 | -10 | 9 | 7 | -2 | -22% |
| 4.3 | 0 | 4 | +4 | 4 | 12 | +8 | 0 | 12 | +12 | +1200% |
| | 4 | 13 | +9 | 13 | 0 | -13 | 4 | 0 | -4 | -100% |
| 5.1 | 7 | 6.5 | -.5 | 6.5 | 12 | +5.5 | 7 | 12 | +5 | +71% |
| | 57 | 3.5 | -53.5 | 3.5 | 3 | -.5 | 57 | 3 | -54 | -95% |
| 6.1 | 2 | 14.5 | +12.5 | 14.5 | 17 | +2.5 | 2 | 17 | +15 | +750% |
| | 52 | 37 | -15 | 37 | 1 | -36 | 52 | 1 | -51 | -98% |
| 6.2 | 1 | 5 | +4 | 5 | 11 | +6 | 1 | 11 | +10 | +1000% |
| | 53 | 42 | -11 | 42 | .5 | -41.5 | 53 | .5 | -52.5 | -99% |
| 6.3 | 1 | 9.5 | +8.5 | 9.5 | 11 | +1.5 | 1 | 11 | +10 | +1000% |
| | 53.5 | 14.5 | -39 | 14.5 | 2 | -12.5 | 53.5 | 2 | -51.5 | -96% |

S = Subject Diff. = Difference in the median number of digits

(A) = baseline (B) = Condition B (lessons 1-10) (C) = Condition C (lessons 11-21)

% of Change: + increase - decrease

Note: Broken line indicates exceptional subjects by classroom.
The number of correct digits is recorded over the number of incorrect digits.

Question 3

When teachers follow the instructional procedures outlined in Multiplication Facts 0 to 81 (Mercer & Miller, 1992b) from the Strategic Math Series, will the performance of individual students with disabilities be similar to the performance of normally achieving peers in the same group on measures of accuracy and fluency? Appendix M contains supporting documentation for the results summarized in Table 28. Pretest scores and digit writing rates on the screening instrument are provided for discussion purposes.

Classroom 4--Subject 4.1. On the pretest, Subject 4.1 scored below the median score of 16 normally achieving subjects (group). Across all lessons completed during Conditions B and C (including the posttest), the accuracy performance of Subject 4.1 remained within the performance range of the group on 79% of the lessons. Scores were at or above the group median level on 47% of the lessons.

On the fluency tool screening, Subject 4.1 wrote more digits per minute than the median number written by 16 normally achieving classmates (group). Across all fluency measures of Conditions B and C, the subject's correct digit writing rate was within the group range on 100% of the measures and at or above the group median score on 40% of the measures.

Classroom 4--Subject 4.2. On the pretest, the score obtained by Subject 4.2 was equal to the median score of 16 normally achieving classmates (group). Across all lessons completed during Conditions B and C (including the posttest), the accuracy performance of Subject 4.2 was within the group range on 100% of the lessons and at or above the group median score on 84% of the lessons.

On the fluency tool screening, Subject 4.2 wrote fewer digits per minute than the median number of digits written by 16 normally achieving classmates (group). Across all fluency measures of Conditions B and C, the subject's correct digit writing rate was within the group range on 100% of the measures. The subject's performance was at or above the group median level on 65% of the measures.

Classroom 4--Subject 4.3. On the pretest, Subject 4.3 scored below the median score of 16 normally achieving classmates (group). Across all lessons completed during Conditions B and C (including the posttest), the accuracy performance of Subject 4.3 was within the group performance range on 89% of the lessons and at or above the group median score on 37% of the lessons.

On the fluency tool skill screening, Subject 4.3 wrote more digits per minute than the median number written by 16 normally achieving classmates (group). Across all fluency measures of Conditions B and C, the subject's correct digit writing rate was within the group range on 81% of the measures. Performance was equal to or above the group median rate on 6% of the measures.

Classroom 5--Subject 5.1. On the pretest, Subject 5.1 scored below the median score of 18 normally achieving classmates (group). Across all lessons completed during Conditions B and C (including the posttest), the accuracy performance of Subject 5.1 was within the group range on 75% of the lessons. The group median score was reached or exceeded on 40% of the lessons.

On the fluency tool skill screening, Subject 5.1 performed well above the median digit writing rate of 18 normally achieving classmates (group). Across all fluency measures of Conditions B and C, the subject's correct digit writing rate was within the group range on 100% of the measures with rates above the group median on 36% of the measures.

Classroom 6--Subject 6.1. On the pretest, Subject 6.1 scored above the median score of 13 normally achieving classmates (group). Across all lessons completed during Conditions B and C (including the posttest), the accuracy performance of Subject 6.1 was within the group range on

100% of the lessons and equal to or above the group median on 70% of the lessons.

On the fluency tool screening, Subject 6.1 wrote fewer digits per minute than the median number of digits written by 13 normally achieving classmates (group). Across all fluency measures of Conditions B and C, the correct digit writing rate of Subject 6.1 was within the group range on 100% of the measures. The subject reached or exceeded the group median rate on 95% of the measures.

Classroom 6--Subject 6.2. On the pretest, Subject 6.2 scored below the median score of 13 normally achieving classmates (group). Across all lessons completed during Conditions B and C (including the posttest), the accuracy performance of Subject 6.2 was within the group range on 94% of the lessons. Performance was equal to or above the group median score on 50% of the lessons.

On the fluency tool skill screening, Subject 6.2 wrote more digits per minute than the median number of digits written by 13 normally achieving classmates (group). Across all fluency measures of Conditions B and C, the subject's correct digit writing rate was within the group range on 100% of the measures with 60% of the rate scores equal to or above the group median rate.

Classroom 6-Subject 6.3. On the pretest, Subject 6.3 obtained a score equal to the median score of 13 normally achieving classmates (group). Across all lessons completed during Conditions B and C (including the posttest), the subject's score was within the group range on 100% of the lessons. The subject's score reached or exceeded the group median score on 53% of the measures.

On the fluency tool screening, Subject 6.3 wrote fewer digits per minute than the median number of digits written by 13 normally achieving classmates (group). Across all fluency measures of Conditions B and C, the subject's correct digit writing rate was within the group range on 100% of the measures with 75% of the rate scores equal to or above the group median rate.

Summary of results for Question 3 (Table 28). On the pretest, 4 exceptional subjects (4.1, 4.3, 5.1, and 6.2) scored below the median score of normally achieving classmates (group). Across all lessons completed during Conditions B and C (including the posttest) accuracy scores achieved by 5 exceptional subjects were within the group range on at least 89% of the lessons. Two exceptional subjects who were below the group median on the pretest stayed within range on 75% (Subject 5.1) and 79% (Subject 4.1) of the lessons. In addition, all exceptional subjects reached or exceeded the group median score during at least 37% of the lessons. Four exceptional subjects (4.2, 6.1, 6.2, and 6.3) had accuracy scores at the group median level or above on half or more of the lessons.

During the fluency tool skill screening, 3 exceptional subjects (4.2, 6.1, and 6.3) wrote fewer digits per minute than the median number of digits written by normally achieving classmates (group). Across all fluency measures of Conditions B and C, correct digit writing rates of all exceptional subjects were within the group range on more than 80% of the measures. Six exceptional subjects (4.1, 4.2, 5.1, 6.1, 6.2, and 6.3) reached or exceeded the group median rate on at least 36% of the measures. Subject 4.3 only reached the group median on 6% of the measures.

## Summary

In this investigation, three experimental questions were asked about the performance of exceptional students (students with disabilities) during large-group instruction in mathematics in mainstream classrooms. Regular classroom teachers followed scripted procedures in Multiplication Facts 0 to 81 (Mercer & Miller, 1992b) from the Strategic Math Series to teach multiplication to their second grade students. Within each second grade class were normally achieving students, students with disabilities, and

Table 28

Individual Performance Compared to Group Performance For Question 3 in Study II

|  | Accuracy |  | Fluency |  |
|---|---|---|---|---|
|  | percentage correct scores (Conditions B and C and Posttest) |  | # of correct digits on timed probes (Conditions B and C) |  |
|  | Percentage of Lessons |  | Percentage of Measures |  |
| S | within group range | group median or above | within group range | group median or above |
| 4.1 | * 79% | 47% | 100% | 40% |
| 4.2 | 100% | 84% | * 100% | 65% |
| 4.3 | * 89% | 37% | 81% | 6% |
| 5.1 | * 75% | 40% | 100% | 36% |
| 6.1 | 100% | 70% | * 100% | 95% |
| 6.2 | * 94% | 50% | 100% | 60% |
| 6.3 | 100% | 53% | * 100% | 75% |

S = Subject

Note: Broken line indicates exceptional subjects by classroom.
Asterisk (*) indicates a score below the group median score on pretest or fluency tool skill screening.

students who failed to demonstrate proficiency in tool and prerequisite skills. The first and second experimental questions were concerned with the effects of instruction under regular classroom conditions on measures of accuracy and fluency for exceptional students. For the third experimental question, the performance of exceptional students was compared to the performance of normally achieving students receiving the same instruction under the same conditions.

Subjects for the investigation were the exceptional and normally achieving students in each classroom who demonstrated acceptable levels of tool and prerequisite skills. Exceptional subjects included 12 students classified as SLD and 1 student classified as EH who were neither accurate nor fluent at solving basic multiplication problems. Normally achieving subjects represented the typical range of performance levels found in mainstream classrooms with the exception of students who were excluded during screening for tool and prerequisite skills. Excluded students received all of the same instruction, but their scores were eliminated from data analysis for the third experimental question in order not to skew results toward lower levels within the class. Results for excluded students are provided in Appendix N and discussed as other findings.

Two studies were conducted simultaneously. For each study, the intervention was introduced into three classrooms within a single subject multiple baseline design. To answer the experimental questions, two dependent variables were monitored: percentage of correct responses on untimed independent practice problems for each consecutive lesson of the curriculum and the number of correct and incorrect digits written per minute on probes administered after each lesson. Baseline levels of responding were collected on the timed probes, and the introduction of instruction into each classroom was controlled through the second consecutive lesson of the curriculum to establish experimental control.

### Question 1

Data for the first experimental question regarding the effects of instruction on accuracy included pretest and posttest scores and scores on independent practice problems of sequential lessons. Results were summarized across both studies in terms of performance during each instructional condition (Table 29).

The first instructional condition (Condition B) included 10 lessons that emphasized understanding of the

Table 29
Summary of Results Across Two Studies For Question 1

| | Condition B | | | | Condition C | |
|---|---|---|---|---|---|---|
| S | # Repeated/ # Completed | Lessons at Criterion | Median % | Increase Pre-Post | Lessons at Criterion | Median % |
| 1.1 | 0/9 | 100% | 90% | 75%* | 0% | 50% |
| 1.2 | 0/9 | 100% | 100% | 60% | 45% | 70% |
| 1.3 | 1/9 | 89% | 90% | 55% | 40% | 70% |
| 2.1 | 0/9 | 100% | 100% | 35% | 64% | 80% |
| 2.2 | 1/9 | 89% | 80% | 25% | 33% | 60% |
| 3.1 | 1/9 | 89% | 100% | 30% | 64% | 80% |

Table 29--Continued

| S | # Repeated/ # Completed | Lessons at Criterion | Median % | Increase Pre-Post | Lessons at Criterion | Median % |
|---|---|---|---|---|---|---|
|  | Condition B |  |  |  | Condition C |  |
| 4.1 | 2/9 | 78% | 90% | 85%* | 67% | 80% |
| 4.2 | 0/9 | 100% | 90% | 80%* | 78% | 100% |
| 4.3 | 1/9 | 89% | 90% | 80%* | 67% | 80% |
| 5.1 | 2/9 | 78% | 90% | 55% | 40% | 60% |
| 6.1 | 1/9 | 89% | 100% | 50% | 100% | 90% |
| 6.2 | 1/8 | 87% | 95% | 70% | 55% | 80% |
| 6.3 | 1/9 | 89% | 90% | 60% | 60% | 80% |

S = Subject * = 90% or above on first posttest attempt

Note: Broken line separates subjects in Studies I and II.

multiplication process using concrete, representational, and abstract procedures. Each lesson (except lesson 7) included a student learning sheet with 10 problems designated for independent practice. A mastery criterion of 80% on independent practice problems controlled movement from one lesson to the next. Teachers were directed to provide additional instruction to students who failed to meet the criterion. Twelve exceptional subjects completed all lessons of Condition B. One subject completed all but one lesson. Across both studies, no exceptional subject scored below the 80% criterion on more than two lessons.

During Condition B, subjects in Study I reached the criterion score on the first attempt on at least 89% of the lessons. In Study II, subjects reached the criterion score on the first attempt on at least 78% of the lessons. In Study I, the median performance level of first attempt scores was 80% or above for all exceptional subjects. In Study II, all median scores were 90% or above.

Examination of results across both studies confirms that all exceptional subjects gained at least 25 percentage points from the pretest to their first attempt on the posttest (between Conditions B and C). The posttest criterion (90%) was reached or exceeded on the first attempt by 4 subjects. Of the other 9 subjects, 7 reached the posttest criterion on a second attempt. One subject required a third attempt to reach the criterion, and 1 subject with a score below the criterion did not retake the posttest.

The second instructional condition (Condition C) included 11 lessons with accompanying learning sheets. During Condition C, independent practice included complex word problems. On several lessons, subjects were also required to write and solve their own word problems. Lessons with below-criterion scores did not require reteaching.

Across both studies, 12 of the 13 subjects reached the criterion level on at least 33% of lessons completed during Condition C. The 1 subject who failed to reach 80% on any lesson was absent frequently and missed instruction on 6 of the 11 lessons. Median performance levels in Study I ranged from 50% to 80% and were lower than in Condition B for all subjects. The effect was similar in Study II. Median performance levels ranged from 60% to 100% and were lower than in the first instructional condition for 6 of the 7 exceptional subjects. One subject had a higher median score during Condition C.

The findings in two studies provide an answer to the first experimental question. Exceptional subjects in second grade were able to reach criterion levels of accurate responding in multiplication during large-group instruction when teachers followed the scripted procedures in Multiplication Facts 0 to 81 (Mercer & Miller, 1992b) from the Strategic Math Series. Percentages of criterion responding were highest when subjects completed lessons sequentially, when a mastery criterion controlled movement within the curriculum, and when independent practice did not include word problems.

### Question 2

Data for the second experimental question regarding the effects of instruction on fluency included the number of correct and incorrect digits written on a 1-minute probe containing abstract multiplication problems. Basic facts from 0 times 0 to 9 times 9 were randomly selected for five versions of the probe that were distributed throughout the investigation. No attempt was made to control for problem difficulty. Rate of performance on the probe was monitored prior to instruction (baseline condition) and following the consecutive lessons of two instructional conditions. Results for the 6 exceptional subjects in study I and the 7 exceptional subjects in Study II were analyzed and summarized within and between conditions.

Within condition analysis included condition length, median performance levels, and performance trends. In addition, data were analyzed for the occurrence of crossover and attainment of the fluency criterion specified in the manual (30 correct digits per minute with 2 or fewer errors).

Between conditions, analysis focused on changes in the median performance, trend, and variability between adjacent conditions. The percentage of overlap between baseline and the first instructional condition was also computed to provide additional information about the effects of instruction.

Within condition analysis showed that, during baseline (Condition A), 5 subjects in Study I were neither accurate nor fluent in multiplication. The number of incorrect digits remained above the number of correct digits throughout the condition. One subject demonstrated some knowledge of multiplication, but incorrect responses were increasing at the same rate as correct responses during baseline. In Study II, all 7 subjects had more incorrect than correct responses during baseline. One subject reduced the total number of responses made on the probes; as a result incorrect responses decreased, but the number of correct responses did not increase. In general, overall performance (correct and incorrect responding) did not improve for any subject during Condition A. Changes in performance when the intervention was introduced are summarized across both studies in Table 30.

Between Conditions A and B, all 6 subjects in Study I increased the median number of correct digits and reduced the number of incorrect digits. In Study II, the effect was replicated for 4 subjects. One subject in Study II reduced the number of both correct and incorrect digits. Two subjects in Study II increased the number of correct and incorrect digits. Across both studies, 12 subjects increased the median number of correct digits, and 11 subjects reduced the median number of incorrect digits when the intervention was introduced into their classroom. Overall median performance (both correct and incorrect responses) improved for 10 subjects.

Trend of performance provided another indicator of change between Condition A (baseline) and Condition B (lessons 1-10). In Study I, correct responses decelerated for 2 subjects, remained level for 2 subjects, and accelerated for 2 subjects in Condition A. During Condition B, corrects accelerated for 5 subjects. One subject, level during Condition A, jumped to a higher level of corrects and stabilized in Condition B. Incorrect

### Table 30
### Summary of Results Between Baseline and Instruction Across Two Studies For Question 2

|  | Median Change (A)-(B) |  | Changes in Trend (A)-(B) (Variability) |  | Percentage of Overlap (A)-(B) |  |
|---|---|---|---|---|---|---|
| S | ● | X | ● | X | ● | X |
| 1.1 | +7 | -47 | L-L(+) | L-D(+) | 0% | 0% |
| 1.2 | +3.5 | -11.5 | L-A(+) | A-D(-) | 11% | 11% |
| 1.3 | +5.5 | -42.5 | D-A(+) | A-D(-) | 20% | 0% |
| 2.1 | +5.5 | -5.5 | A-A(+) | A-D(=) | 10% | 30% |
| 2.2 | +2 | -9.5 | D-A(+) | A-D(+) | 30% | 70% |
| 3.1 | +1 | -1.5 | A-A(+) | A-D(+) | 50% | 70% |

Table 30--Continued

| S | Median Change (A)-(B) |  | Changes in Trend (A)-(B) (Variability) |  | Percentage of Overlap (A)-(B) |  |
|---|---|---|---|---|---|---|
|  | ● | X | ● | X | ● | X |
| 4.1 | +6.5 | -1 | A-A(+) | A-D(+) | 11% | 100% |
| 4.2 | +12 | +8 | L-A(+) | D-A(+) | 0% | 55% |
| 4.3 | +4 | +9 | NED | NED | NED | NED |
| 5.1 | -.5 | -53.5 | A-A(-) | A-D(-) | 50% | 0% |
| 6.1 | +12.5 | -15 | D-A(+) | A-D(-) | 10% | 90% |
| 6.2 | +4 | -11 | D-A(+) | A-D(+) | 33% | 55% |
| 6.3 | +8.5 | -39 | D-A(+) | A-D(=) | 0% | 10% |

S = Subject (A) = Condition A (baseline) (B) = Condition B (lessons 1-10)
● = correct digits X = incorrect digits
NED = not enough data points
L = level trend A = accelerating trend D = decelerating trend
+ increased variability - decreased variability = equal variability

responses changed from accelerating or high level trends prior to instruction to decelerating trends during the first instructional condition for all subjects.

In Study II, trend lines were analyzed between Conditions A and B for 6 of the 7 subjects. During Condition A, trends for correct responses decelerated for 3 subjects, remained level for 1 subject, and accelerated for 2 subjects. During Condition B, correct responses accelerated for all subjects. Incorrect responses changed from accelerating trends prior to instruction to decelerating trends during the first instructional condition. One subject reversed a decelerating baseline trend for incorrects during the first instructional condition. Across both studies, the trend of performance improved for 11 subjects when the intervention was introduced.

Variability of responding was defined as performance fluctuation around the trend lines. In Study I, variability of correct responding increased for all subjects from baseline to the first instructional condition. Variability of incorrect responding increased for 3 subjects, remained the same for 1 subject, and decreased for 2 subjects. Similar results occurred in Study II. Variability of correct responding increased for 5 of the 6 subjects with performance trends in the baseline condition. One subject showed a decrease in variability. Variability of incorrect responses increased for 3 subjects, remained the same for 1 subject, and decreased for 2 subjects. Across both studies, performance on writing correct and incorrect digits improved but was highly unstable during the first instructional condition for all of the subjects with baseline trends.

Percentages of response overlap were calculated for both correct and incorrect digits between conditions with and without instruction (Conditions A and B). Lower percentages of overlap indicate more treatment effect. Since no firm criteria have been established, criteria for this study were based on the median percentages of overlap in the single subject studies reviewed in Chapter II--29% for correct digits and 20% for incorrect digits. In Study I, 4 subjects were below the 29% criteria for correct digits; 3 subjects were below the 20% criteria for incorrect digits. In Study II, 4 subjects were below the criterion for correct digits. Two subjects were below the criterion for incorrect digits. Across both studies, 8 subjects were below the criterion for correct digits, and 5 subjects were below the criterion for incorrect digits. Four subjects were below the criteria for both correct and incorrect digits.

Results between the two instructional conditions are summarized across two studies in Table 31. During the first 10 lessons (Condition B), 3 of the 5 subjects in Study I and 2 of the 7 subjects subjects in Study II who wrote more incorrect than correct digits during baseline demonstrated crossover (more corrects than incorrects) and maintained the pattern for the remainder of instruction. The other subjects reached crossover during the second instructional condition when fluency practice with feedback became part of the instructional procedures (Condition C). Across both studies, all subjects demonstrated crossover by the end of the investigation.

In Study I, the subject with prior knowledge of multiplication reached criterion responding (30 correct digits with 2 or fewer errors) on a single day. Three other subjects failed to reach the 30 correct digits but did reduce the number of incorrect digits to 2 or fewer per minute. Two subjects failed to reach either criterion by the end of the study. In Study II, 5 subjects failed to reach the 30 correct digits but did reach the criterion for incorrect digits. Two subjects failed to reach either criterion by the end of the study. Across both studies, 1 subject reached the criterion for correct digits, and 9 subjects reached the criterion for incorrect digits.

Table 31

Summary of Results of Instruction Across Two Studies For Question 2

| | Crossover Lesson | Criterion Responding | | Median Change (B)-(C) | | Changes in Trend (B)-(C) (Variability) | | Overall Percentage of Change | |
|---|---|---|---|---|---|---|---|---|---|
| S | # | ● | X | ● | X | ● | X | ● | X |
| 1.1 | 7 | N | Y | +1 | -7 | L-A(-) | D-L(-) | +800% | -100% |
| 1.2 | 12 | N | Y | +6 | -3 | A-A(+) | D-D(-) | +633% | -94% |
| 1.3 | 2 | N | N | +8 | +1 | A-L(+) | D-D(-) | +1350% | -94% |
| 2.1 | 10 | N | Y | +2.5 | -1.5 | A-A(=) | D-D(-) | +400% | -78% |
| 2.2 | 18 | N | N | +1 | +1.5 | A-A(=) | D-L(-) | +150% | -57% |
| 3.1 | baseline | Y | Y | +7.5 | -1.5 | A-A(+) | D-L(-) | +80% | -75% |

Table 31--Continued

| | Crossover Lesson | Criterion Responding | | Median Change (B)-(C) | | Changes in Trend (B)-(C) (Variability) | | Overall Percentage of Change | |
|---|---|---|---|---|---|---|---|---|---|
| S | # | ● | X | ● | X | ● | X | ● | X |
| 4.1 | 5 | N | Y | +5 | -3 | A-D(-) | D-D(=) | +766% | -66% |
| 4.2 | 13 | N | N | +6 | -10 | A-A(+) | A-D(-) | +1800% | -22% |
| 4.3 | 13 | N | Y | +8 | -13 | A-A(+) | A-L(-) | +1200% | -100% |
| 5.1 | 19 | N | N | +5.5 | -.5 | A-A(+) | D-D(=) | +71% | -95% |
| 6.1 | 9 | N | Y | +2.5 | -36 | A-A(=) | D-D(-) | +750% | -98% |
| 6.2 | 12 | N | Y | +6 | -41.5 | A-D(-) | D-D(-) | +1000% | -99% |
| 6.3 | 15 | N | Y | +1.5 | -12.5 | A-A(=) | D-A(-) | +1000% | -96% |

S = Subject (B) = Condition B (lessons 1-10) (C) = Condition C (lessons 11-21)
● = correct digits X = incorrect digits
Y = yes N = no
L = level trend A = accelerating trend D = decelerating trend
+ increase - decrease = no change

Note: Broken line separates subjects in Studies I and II.

In Study I, all subjects increased the median number of correct digits between instructional conditions. Four subjects also reduced the median number of incorrect digits. Incorrect digits increased slightly for 2 subjects. In Study II, all subjects increased the median number of correct digits and reduced the median number of incorrect digits. Across both studies, 11 subjects showed improvement for both correct and incorrect digits when fluency practice with feedback became part of instruction.

In Study I, 5 subjects ended the investigation with correct responses that were accelerating. One subject's correct responses had leveled off below the criterion level. Incorrect responses were level or still decelerating for all subjects. In Study II, 5 subjects ended with accelerating correct responses. Corrects were decelerating for 2 subjects. Decelerating incorrect responses were noted for 5 subjects. Incorrect responses were level for 1 subject and accelerating slightly for 1 subject. Across both studies, correct responses were still accelerating while incorrect responses were decelerating or had leveled off below the criterion for 8 subjects. The other 5 subjects either had decelerating incorrects with level corrects, accelerating corrects with level or slightly accelerating incorrects, or decelerating trends for both correct and incorrect responses.

With a shift in the instructional focus between conditions, changes were observed in the variability of responding. Across both studies, ending variability indicated that correct responding was still somewhat unstable for 6 subjects. Variability for incorrect responding decreased between instructional conditions for 11 subjects. The other 2 subjects showed the same amount of variability between instructional conditions.

From the beginning to the end of the investigation, all subjects increased the number of correct digits and reduced the number of incorrect digits. In Study I, percentages of change ranged from 80% to 1350% for the number of correct digits and from 57% to 100% for the number of incorrect digits. In Study II, the percentage of change for corrects ranged from 71% to 1800%. For incorrects, percentages of change ranged from 22% to 100%. Across both studies, 11 subjects increased the number of correct digits by 150% or more. The number of incorrect digits was reduced by more than 90% by 8 subjects.

The findings in two studies provide an answer to the second experimental question. In general, exceptional subjects in second grade did not reach the criterion for the number of correct digits during large-group instruction using Multiplication Facts 0 to 81 (Mercer & Miller, 1992b) from the Strategic Math Series. However, most subjects did reduce the number of incorrect digits below the error criterion.

Other effects on fluency were observed. All subjects demonstrated and maintained crossover before the end of instruction, and based on median levels and overall percentages of change, all subjects increased the number of correct digits and reduced the number of incorrect digits during instruction. Based on trends in the final condition, performance was still improving at the end of the investigation for 8 of 13 subjects. During both instructional conditions, incorrect responding was more stable than correct responding.

### Question 3

Analyses for the first two experimental questions confirm that effects on both accuracy and fluency were observed for exceptional subjects during large-group instruction using Multiplication Facts 0 to 81 (Mercer & Miller, 1992b) from the Strategic Math Series. How did the effects for exceptional subjects compare to the effects for normally achieving subjects who received the same instruction under the same conditions? That question was investigated through analysis of the percentage scores (accuracy) and correct digit writing rates (fluency) demonstrated by both exceptional and normally achieving subjects within each classroom. The percentages of lessons and measures on which exceptional subjects scored within the range and at or above the median level obtained by normally achieving classmates are displayed for six classrooms across two studies in Table 32.

Across all lessons completed during Conditions B and C (including the posttest), all of the exceptional subjects in Study I obtained scores within the performance range demonstrated by normally achieving classmates on at least 84% of the lessons. In addition, all exceptional subjects scored at or above the group median score on at least 16% of the lessons. In Study II, 5 exceptional subjects scored within range on at least 89% of completed lessons. The other 2 subjects were within range on 79% and 75% of completed lessons. Subjects in Study II scored at or above the group median score on at least 37% of the lessons.

Across all fluency measures of Conditions B and C, correct digit writing rates for all exceptional subjects in Study I were within the range of digits written by normally achieving classmates (group) on at least 84% of the measures. Five subjects reached the normally achieving group median score on at least 31% of the measures. Only

Table 32

Summary of Results Across Two Studies For Question 3

| S | Accuracy: percentage correct scores (Conditions B and C and Posttest) | | Fluency: # of correct digits on timed probes (Conditions B and C) | |
|---|---|---|---|---|
| | Percentage of Lessons | | Percentage of Measures | |
| | within group range | group median or above | within group range | group median or above |
| 1.1 | 93% | 33% | * 84% | 31% |
| 1.2 | * 100% | 62% | * 89% | 33% |
| 1.3 | * 100% | 50% | * 100% | 45% |
| 2.1 | 90% | 52% | * 100% | 43% |
| 2.2 | * 84% | 16% | * 84% | 0% |
| 3.1 | 95% | 62% | 100% | 90% |

Table 32--Continued

| | Accuracy | | Fluency | |
|---|---|---|---|---|
| | percentage correct scores (Conditions B and C and Posttest) | | # of correct digits on timed probes (Conditions B and C) | |
| | Percentage of Lessons | | Percentage of Measures | |
| S | within group range | group median or above | within group range | group median or above |
| 4.1 | * 79% | 47% | 100% | 40% |
| 4.2 | 100% | 84% | * 100% | 65% |
| 4.3 | * 89% | 37% | 81% | 6% |
| 5.1 | * 75% | 40% | 100% | 36% |
| 6.1 | 100% | 70% | * 100% | 95% |
| 6.2 | * 94% | 50% | 100% | 60% |
| 6.3 | 100% | 53% | * 100% | 75% |

S = Subject

Note: Broken line separates subjects in Studies I and II.
Asterisk (*) indicates a score below the group median score on pretest or fluency tool skill screening.

1 subject failed to reach the group median score on any fluency measure. All subjects in Study II performed within the group range on at least 81% of the measures. Six subjects reached or exceeded the group median score on at least 36% of the measures. The group median score was reached by 1 subject on only 6% of the measures.

Performance within the range of that demonstrated by normally achieving classmates on 80% of completed lessons was the criterion for validation of the curriculum as an appropriate intervention for large-group mathematics instruction in regular classrooms with mainstreamed exceptional students. Across both studies, the within-range criterion was reached on measures of accuracy by 11 of the 13 exceptional subjects. On measures of fluency, the within-range criterion was reached by all 13 exceptional subjects. In addition, exceptional subjects did not always have the lowest scores in the class. All of the exceptional subjects were able to reach the group median score on measures of accuracy, and 12 of the 13 subjects reached the group median score on measures of fluency. The curriculum was clearly appropriate as a large-group intervention in regular classrooms with mainstreamed exceptional subjects.

To further examine the overall effects of instruction, median performance levels of exceptional subjects during four phases of instruction (concrete, representational, abstract, and word problem/fluency practice) were compared to median performance levels of normally achieving subjects during the same phases. For overall effects on accuracy across both studies, the median score for each subject during each phase was calculated using subjects' first scores on the learning sheets (raw data found in Appendices J and K). Median scores for each phase were categorized as below the 80% mastery criterion or between 80% and 100%. For each phase, median scores were reported for subjects with at least two scores during the phase. Results are summarized in Figure 21.

During the concrete phase of instruction (lessons 1-3), 7 normally achieving subjects were below the 80% mastery criterion. One exceptional subject was below the mastery criterion. The remaining 92 normally achieving subjects and 12 exceptional subjects achieved a median score between 80% and 100%.

For the three lessons in the representational phase (lessons 4-6), only 1 normally achieving subject had a median score below the 80% mastery criterion. The other

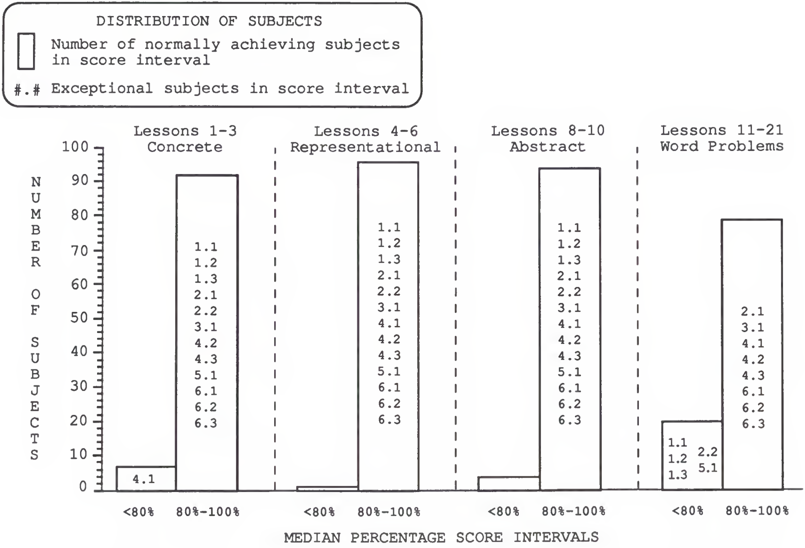


Figure 21. Distribution of median percentage scores across two studies.

96 normally achieving subjects and all exceptional subjects showed a median score between 80% and 100%.

During the abstract phase (lessons 8-10), 4 normally achieving subjects were below the 80% mastery criterion. The other 94 normally achieving subjects and all exceptional subjects achieved a median score between 80% and 100%.

For the lessons that included word problems (lessons 11-21), 20 normally achieving subjects and 5 exceptional subjects had a median score below the 80% criterion. Seventy-nine normally achieving subjects and 8 exceptional subjects obtained a median score between 80% and 100%.

The percentages of subjects in each category are presented visually in Figure 22. Visual inspection confirms that, as a group, exceptional subjects performed similarly to the normally achieving group of subjects during the concrete, representational, and abstract phases of instruction. During the word problem phase, performance for the two groups was different. A higher percentage of exceptional subjects had median scores in the below 80% category. However, the majority of exceptional subjects (61.5%) had a median score between 80% and 100%.

For overall effects on fluency across both studies, the median score for each subject during each phase was

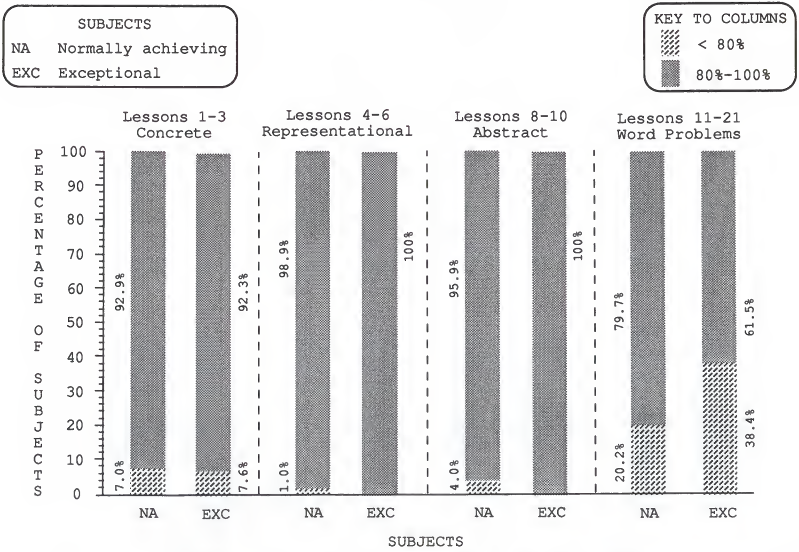


Figure 22. Comparison of median percentage scores for normally achieving and exceptional subjects across two studies.

calculated using the number of correct digits written per minute on abstract probes (raw data found in Appendices J and K). Median scores for each phase were categorized as between 0 and 9.5 correct digits per minute, between 10 and 19.5 correct digits per minute, or more than 20 correct digits per minute. Median scores were not calculated for subjects with fewer than two scores during any phase. Results are summarized in Figure 23.

During the concrete phase of instruction (lessons 1-3), 78 normally achieving subjects and 11 exceptional subjects wrote between 0 and 9.5 correct digits per minute. Sixteen normally achieving subjects and 2 exceptional subjects were in the 10 to 19.5 correct digits per minute category. No subject wrote more than 20 correct digits per minute.

Across the three lessons of the representational phase of instruction (lessons 4-6), 62 normally achieving subjects and 8 exceptional subjects wrote between 0 and 9.5 correct digits per minute. Thirty-one normally achieving subjects and 4 exceptional subjects wrote between 10 and 19.5 correct digits per minute. One normally achieving subject wrote more than 20 correct digits per minute.

During the abstract phase (lessons 7-10), 59 normally achieving subjects and 7 exceptional subjects wrote between 0 and 9.5 correct digits per minute. Fewer subjects (38

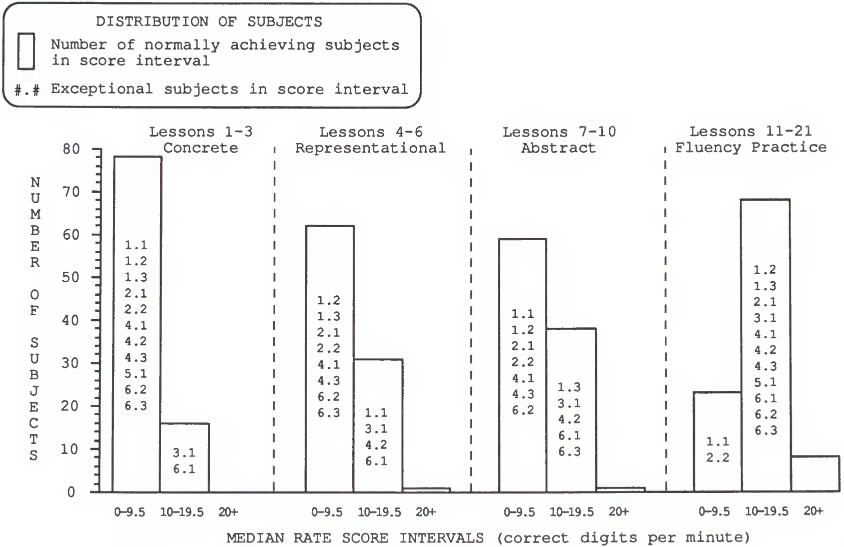


Figure 23. Distribution of median rate scores across two studies.

normally achieving and 5 exceptional) reached the 10-19.5 category. One normally achieving subject wrote more than 20 correct digits per minute.

During the fluency practice phase (lessons 11-21), only 23 normally achieving subjects and 2 exceptional subjects were still in the lowest category of correct digits per minute (0-9.5). Most of the subjects (68 normally achieving and 11 exceptional) wrote between 10 and 19.5 correct digits per minute. Eight normally achieving subjects wrote more than 20 correct digits per minute.

The percentages of subjects in each category are presented visually in Figure 24. Visual inspection confirms that, as a group, exceptional subjects performed similarly to the normally achieving group of subjects during all phases of instruction. Two differences occurred during the fluency practice phase. A higher percentage of exceptional subjects wrote between 10 and 19.5 correct digits per minute, but 8% of the normally achieving subjects reached the 20 or more correct digits per minute category.

## Other Findings

In the third experimental question of this investigation, the performance of exceptional subjects was compared to the performance of normally achieving subjects

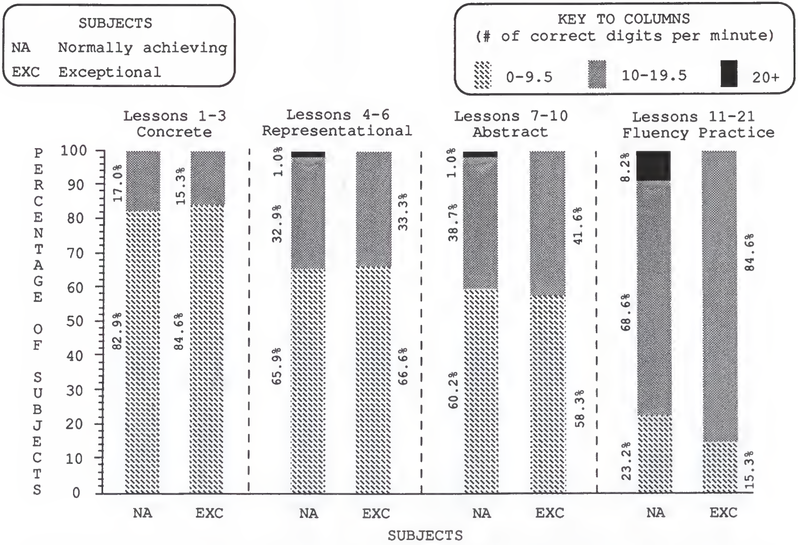


Figure 24. Comparison of median rate scores (correct digits per minute) for normally achieving and exceptional subjects across two studies.

who received the same instruction under the same classroom conditions. Prior to the beginning of instruction, all class members were screened for tool and prerequisite skills. Students who wrote fewer than 30 digits per minute and students who scored below 70% on counting to 81 and solving basic addition problems were designated as excluded students. Excluded students received the same instruction as all other students but, in order to avoid comparing exceptional subjects' performance to the lowest performance levels in the class, excluded students' scores were eliminated from the range and median summaries for each class.

Seven students were excluded for writing fewer than 30 digits per minute (fluency tool skill). Four students were excluded for scoring below 70% on basic addition problems (prerequisite skill for multiplication). Percentage correct scores on each lesson of the curriculum and rate measures on the probes (correct and incorrect digits per minute) are provided for excluded students in each classroom in Appendix N. Excluded students' performance compared to the performance of normally achieving students with proficient levels of tool and prerequisite skills is summarized by classroom in Table 33.

Table 33

Performance of Excluded Students Across Two Studies

|  | Accuracy |  | Fluency |  |
|---|---|---|---|---|
|  | percentage correct scores (Conditions B and C and Posttest) |  | # of correct digits on timed probes (Conditions B and C) |  |
|  | Percentage of Lessons |  | Percentage of Measures |  |
| S | within group range | group median or above | within group range | group median or above |
| 1A | 80% | 35% | DWR 94% | 44% |
| 1B | (+) 69% | 25% | 59% | 0% |
| 1C | 100% | 76% | DWR 100% | 80% |
| 1D | 100% | 57% | DWR 84% | 16% |
| 3A | 100% | 95% | DWR 100% | 62% |
| 3B | 63% | 5% | DWR 52% | 0% |
| 3C | 95% | 52% | DWR 78% | 5% |
| 3D | (+) 100% | 55% | 90% | 19% |

Table 33--Continued

| | Accuracy | | Fluency | |
|---|---|---|---|---|
| | percentage correct scores (Conditions B and C and Posttest) | | # of correct digits on timed probes (Conditions B and C) | |
| | Percentage of Lessons | | Percentage of Measures | |
| S | within group range | group median or above | within group range | group median or above |
| 4A | 87% | 13% | DWR 65% | 0% |
| 4B | (+) 74% | 10% | 100% | 53% |
| 6A | (+) 100% | 76% | 100% | 48% |

S = excluded student
(+) = excluded on basis of solving addition problems
DWR = excluded on basis of digit writing rate
Note: Broken line separates excluded students by classroom.

Unexpectedly, students who had difficulty solving basic addition problems scored within range on untimed multiplication problems (accuracy measures) on at least 63% of completed lessons. Even more surprising is that scores reached or exceeded the group median score on some lessons for all excluded students. Five of the excluded students were within range on all lessons and reached the group median score on more than half of the lessons.

On timed measures of fluency, excluded students who wrote fewer than 30 digits per minute were able to stay within the normally achieving group range on at least 52% of the measures. Eight students reached the group median score on some measures. Examination of the raw data in Appendix N shows that all excluded students reached crossover by the end of the study.

To further examine the overall effects of instruction, median performance levels of excluded students during the concrete, representational, abstract, and word problem/ fluency practice phases of instruction were compared to median performance levels of normally achieving students during the same phases. For overall effects on accuracy across both studies, the median score for each student during each phase was calculated using students' first scores on the learning sheets (raw data found in Appendices

K and N). Median scores for each phase were categorized as below the 80% mastery criterion or between 80% and 100%. Median scores were not calculated for students with fewer than two scores during any phase. Results are summarized in Figure 25.

During the concrete phase of instruction (lessons 1-3), 7 normally achieving students were below the 80% mastery criterion. Two excluded students were below the mastery criterion. The remaining 92 normally achieving subjects and 9 excluded students achieved a median score between 80% and 100%.

For the three lessons in the representational phase (lessons 4-6), 1 normally achieving student and 3 excluded students had median scores below the 80% mastery criterion. The other 96 normally achieving subjects and 8 excluded students showed a median score between 80% and 100%.

During the abstract phase (lessons 8-10), 4 normally achieving students and 3 excluded students were below the 80% mastery criterion. The other 94 normally achieving students and 8 excluded students achieved a median score between 80% and 100%.

For the lessons that included word problems (lessons 11-21), 20 normally achieving students and 3 excluded students had a median score below the 80% criterion.

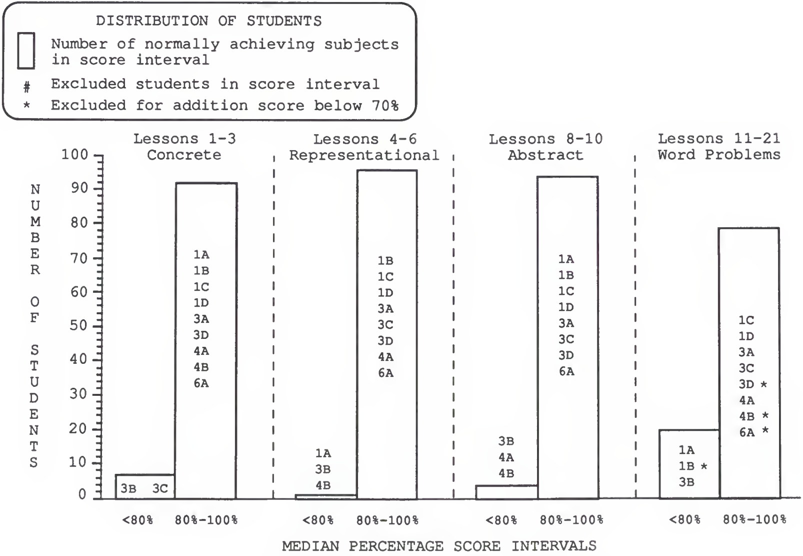


Figure 25. Distribution of median percentage scores across two studies.

Seventy-nine normally achieving subjects and 8 excluded students obtained a median score between 80% and 100%. Among the 8 excluded students who obtained a median score between 80% and 100% were 3 students who scored below 70% on solving basic addition problems during screening.

The percentages of subjects in each category are presented visually in Figure 26. Visual inspection confirms that, as a group, excluded students received lower scores than normally achieving students during the concrete, representational, and abstract phases of instruction. Performance of the two groups was fairly similar during the word problem phase of instruction.

For overall effects on fluency across both studies, the median score for each student during each phase was calculated using the number of correct digits written per minute on abstract probes (raw data found in Appendices K and N). Median scores for each phase were categorized as between 0 and 9.5 correct digits per minute, between 10 and 19.5 correct digits per minute, or more than 20 correct digits per minute. Median scores were not calculated for subjects with fewer than two scores during any phase. Results are summarized in Figure 27.

During the concrete phase of instruction (lessons 1-3), 78 normally achieving students and 10 excluded students

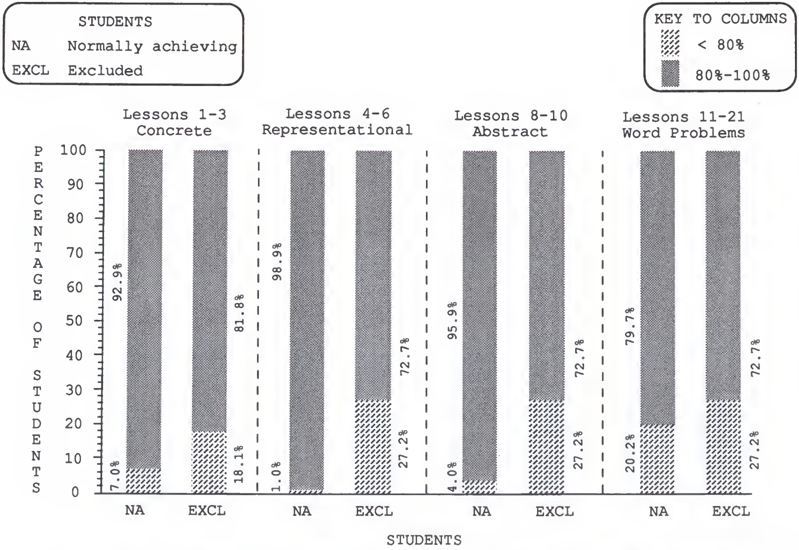


Figure 26. Comparison of median percentage scores for normally achieving and excluded students across two studies.

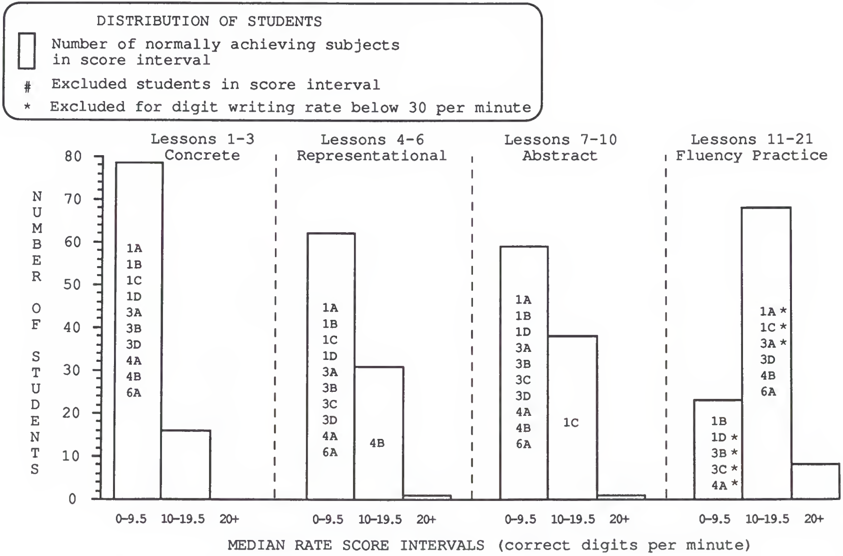


Figure 27. Distribution of median rate scores across two studies.

wrote between 0 and 9.5 correct digits per minute. Sixteen normally achieving students were in the 10 to 19.5 correct digits per minute category. No student wrote more than 20 correct digits per minute.

Across the three lessons of the representational phase of instruction (lessons 4-6), 62 normally achieving students and 10 excluded students wrote between 0 and 9.5 correct digits per minute. Thirty-one normally achieving student and 1 excluded student wrote between 10 and 19.5 correct digits per minute. One normally achieving student wrote more than 20 correct digits per minute.

During the abstract phase (lessons 7-10), 59 normally achieving students and 10 excluded students wrote between 0 and 9.5 correct digits per minute. Fewer students (38 normally achieving and 1 excluded) reached the 10-19.5 category. One normally achieving student wrote more than 20 correct digits per minute.

During the fluency practice phase (lessons 11-21), 23 normally achieving students and 5 excluded students were still in the lowest category of correct digits per minute (0-9.5). Sixty-eight normally achieving students and 6 excluded students wrote between 10 and 19.5 correct digits per minute. Among the 6 excluded students in the 10-19.5 category were 3 students who wrote fewer than the required

30 digits per minute during screening. Eight normally achieving students wrote more than 20 correct digits per minute.

The percentages of subjects in each category are presented visually in Figure 28. Visual inspection confirms that, as a group, excluded students wrote fewer correct digits per minute during all phases of instruction. Although lower than the percentage of normally achieving students in the same category, 54.5% of the excluded students reached 10-19.5 correct digits per minute during the fluency practice phase of instruction.

The findings for excluded students further validate use of Multiplication Facts 0 to 81 (Mercer & Miller, 1992b) from the Strategic Math Series for large-group instruction in multiplication at the elementary level. Students who were expected to have the lowest scores in the class (based on screening results) performed similarly to their more proficient classmates in terms of range and median scores. The unexpected finding that students who had difficulty with addition were able to demonstrate mastery of multiplication requires further investigation. The finding that students who wrote the fewest number of digits during screening reached the same fluency level as the majority of more proficient classmates is an additional area of interest.

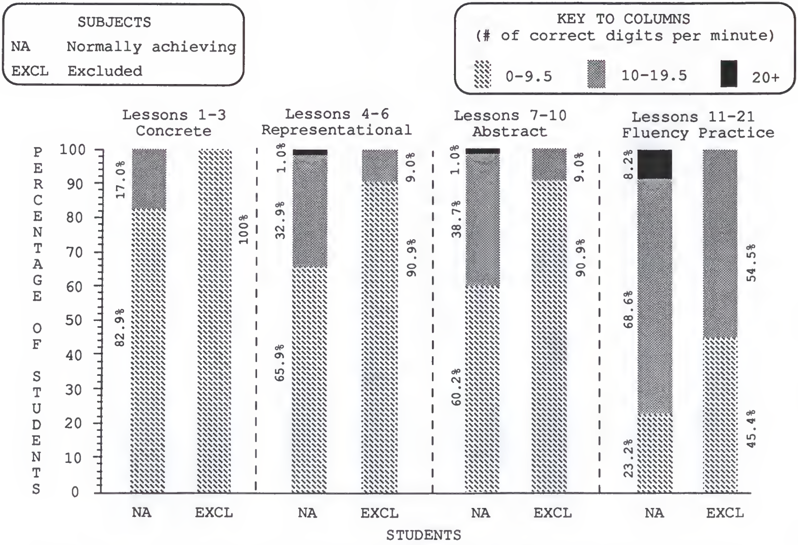


Figure 28. Comparison of median rate scores (correct digits per minute) for normally achieving and excluded students across two studies.

# CHAPTER V
# DISCUSSION

The purpose of this chapter is to present a discussion of the results of two studies designed to investigate the effects of using a scripted, structured curriculum to teach multiplication to large instructional groups that included mainstreamed elementary students with disabilities. In both studies, the effects of instruction on computational accuracy and fluency were investigated as well as the levels of performance reached by students with disabilities compared to levels of performance demonstrated by peers without disabilities in the same instructional groups.

To facilitate discussion, the chapter is divided into four major sections. First, the purpose of the study and supporting literature are reviewed. Next, the methods used in the investigation are described. In the third section, results are discussed in terms of the three experimental questions. In addition, limitations on the interpretation of results are described. Finally, implications of the findings in this investigation and suggestions for future research are presented.

## Review of the Purpose and Supporting Literature

In the American system of education, a subsystem known as special education has been developed to meet the needs of students who fail to succeed in traditional classroom environments due to physical, mental, emotional, or learning disabilities. Historically, following testing for identification of the disability, special education students were removed from their general education classrooms and placed in more controlled settings where specialist teachers provided individualized educational services. The benefits of segregated placements have been questioned, and currently a more inclusive perspective is being recommended for students with disabilities. Current placement options include mainstreaming arrangements in which students with disabilities remain in regular classroom settings for most, if not all, of their academic instruction.

Unfortunately, many of the instructional and curricular traditions of general education make it difficult for regular classroom teachers to provide the type of instruction known to be effective for special education students. As a result, mainstreamed students with disabilities continue to struggle academically and stay "increasingly out of step with the curriculum and grade-level standards of regular education" (Cawley et al., 1992, p. 41).

Classroom conditions can be improved through teachers' use of instructional behaviors associated with student success. Effective teaching behaviors for general academic learning have been identified by numerous researchers in both regular and special education.

Mercer and Miller (1992c) conducted an extensive review of the literature and extended the research on effective teaching to the area of mathematics. According to Mercer and Miller, best practices for teaching mathematics include (a) providing systematic and explicit instruction, (b) teaching understanding of mathematical concepts, (c) providing feedback, (d) teaching generalization, (e) teaching to mastery levels, (f) setting specific goals and expectancies, (g) teaching strategies for problem solving, (h) monitoring progress, (i) promoting positive attitudes toward mathematics, and (j) selecting appropriate content.

The basal curricula typically used for elementary mathematics instruction incorporate few of the features identified by Mercer and Miller (1992c). Cawley and his colleagues (1992) have challenged educators to "modify the way we teach mathematics so it offers the child a different way to learn" (p. 41). Dixon and Carnine (1990) urge professionals to "demand empirically validated approaches that take into account not only the design of the curriculum, but the way the content is to be communicated and is to be implemented by teachers" (p. 1).

The Strategic Math Series is a recently developed, research-based curriculum for students with learning problems that integrates the critical features of effective mathematics instruction for mastery of basic computational facts. Scripted teaching manuals have been field tested for addition (Miller & Mercer, 1991a), subtraction (Miller & Mercer, 1991b), multiplication (Mercer & Miller, 1992b), and division (Mercer & Miller, 1992a).

The teaching manuals of the Strategic Math Series follow a consistent format. For each mathematical operation, students demonstrate their understanding of the process and practice the skill within a controlled sequence of increasingly abstract levels of application. In the beginning, students solve problems using concrete objects. At the next level in the instructional sequence, drawings are used to represent and solve problems. At the most abstract level, students use only numerical symbols to solve problems. Based on the principles of strategic learning, students learn a mnemonic strategy that cues independent solution of unfamiliar problems at the abstract level. Scripted teaching procedures are explicit and systematic for all instructional activities. Other features of effective instruction incorporated within the design of the curriculum include specified performance goals, programmed fluency practice, an elaborated feedback routine, and generalization training.

Sequenced learning in which students first experience a mathematical concept with concrete objects, then drawings, and finally abstract symbols derives primarily from cognitive theories that human understanding occurs at hierarchical levels (Bruner, 1966; Piaget, 1960; Reisman, 1982; Underhill, 1977; Underhill et al., 1980). The behavioral theory that prerequisite skills must be identified through detailed task analysis and mastered prior to new learning generates a similar teaching approach (Gagné, 1977). The effectiveness of the concrete to representational to abstract sequence for mathematical learning has been reliably demonstrated by a number of investigators (Avino, 1990; Clifton, 1990; Essinger, 1988; Furtado, 1989; Gayler, 1987; Kelly, 1986; Matson, 1988; Natzke, 1989; Peterson, 1987/1988; Reid, 1988; Robertson, 1986; Sealander, 1990/1991; Sigda, 1983; Wiers, 1990).

Cognitive theories related to knowledge construction, inner conversation, motivation, and memory provide the foundation for the Learning Strategies Curriculum developed by researchers at the University of Kansas Institute for Research in Learning Disabilities (Deshler & Lenz, 1989; Deshler et al., 1982; Deshler et al., 1984; Schumaker et al., 1983). Learning strategies, the "techniques, principles, or rules that enable a student to learn, to solve problems, and to complete tasks independently" (Deshler & Schumaker, 1986, p. 583), include memorization and application of a mnemonic cue directly related to the problem-solving action required by a task. The effectiveness of embedding a mnemonic cue within the C-R-A sequence in mathematics was demonstrated by Natzke (1989), Clifton (1990), and Wiers (1990).

The benefits of explicit, teacher-directed instruction and controlled practice opportunities are described in the effective teaching research (e.g. Rosenshine & Stevens, 1986). Positive student outcomes have been consistently reported for the teaching procedures and curriculum materials based on the Direct Instruction Model developed at the University of Oregon (Bereiter & Engelmann, 1966; Carnine et al., 1990; Engelmann & Carnine, 1982; Gersten et al., 1987). Several investigators (Carnine & Stein, 1981; Cook & Dossey, 1982; Kameenui et al., 1986; Lloyd et al., 1981; Thornton, 1978, 1990) have demonstrated the effectiveness of using explicit and systematic procedures to teach a solution strategy for mathematical computation.

The importance of setting unambiguous goals and communicating expectations to students appears throughout the effective teaching research (e.g. Christenson et al., 1989). Goal-setting investigators (Locke & Latham, 1990; Locke et al., 1981) found that setting challenging goals results in greater effort and achievement than setting goals that are easily reached. Active student involvement in setting goals for learning mastery fits within the cognitive framework of human learning.

Allowing sufficient instructional time for students to master new learning is another characteristic of effective teaching. The rate at which an individual performs a task has been documented by behavioral researchers as a reliable indicator of skill mastery (Hasselbring et al., 1987; Kirby & Becker, 1988; Lovitt, 1989).

The use of corrective feedback has been associated with improved student achievement (Gersten et al., 1987). In addition to achievement, Wang (1987) described several other desirable outcomes from the appropriate use of feedback; outcomes included active student involvement, independent learning, and motivation for further learning. Incorporating a cognitive approach, Kline et al. (1991)

found that, for students with learning disabilities, instructional efficiency could be improved by using an elaborated feedback routine that includes error analysis and correction procedures.

Frequent, active monitoring of student performance is another essential feature of effective teaching (Good & Brophy, 1986). Use of behavioral data to plan or modify instruction and provide appropriate feedback is associated with improved student learning (Christenson et al., 1989; Fuchs, 1986; Slavin & Madden, 1989).

Generalization refers to the occurrence of a trained behavior under different conditions than those that were in effect during training (Cooper et al., 1987). Early researchers found that generalization rarely occurred when it was treated as a passive behavioral phenomenon (Stokes & Baer, 1977). Cognitive researchers Ellis et al. (1987a, 1987b) confirmed that generalization activities must be an integral part of the instructional process.

The Strategic Math Series represents an integration of cognitive and behavioral learning theories as well as theories related to curriculum design. The influence of theory and research on specific instructional components of the series is summarized in Table 34.

Table 34

Theoretical Foundation For Instructional Components of The Strategic Math Series

| Instructional Component | Major Theoretical Foundation For Instructional Component | Supporting Literature |
|---|---|---|
| + C-R-A sequence | COGNITIVE | Bruner (1966)<br>Gagné (1977)<br>Piaget (1960)<br>Reisman (1982)<br>Underhill (1977)<br>Underhill et al. (1980) |
| + strategy instruction | COGNITIVE | Deshler & Lenz (1989)<br>Deshler & Schumaker (1986)<br>Deshler et al. (1982)<br>Deshler et al. (1984)<br>Schumaker et al. (1983) |
| + explicit, scripted instruction and controlled practice opportunities | CURRICULUM DESIGN | Bereiter & Engelmann (1966)<br>Carnine et al. (1990)<br>Engelmann & Carnine (1982)<br>Gersten et al. (1987)<br>Rosenshine & Stevens (1986) |

Table 34--Continued

| Instructional Component | Major Theoretical Foundation For Instructional Component | Supporting Literature |
|---|---|---|
| + student commitment for meeting goals<br>+ mastery criterion | COGNITIVE<br>BEHAVIORAL | Hasselbring et al. (1987)<br>Kirby & Becker (1988)<br>Locke & Latham (1990)<br>Locke et al. (1981)<br>Lovitt (1989) |
| + elaborated feedback routine | COGNITIVE | Gersten et al. (1987)<br>Kline et al. (1991)<br>Wang (1987) |
| + frequent, data based monitoring | BEHAVIORAL | Christenson et al. (1989)<br>Fuchs (1986)<br>Good & Brophy (1986)<br>Slavin & Madden (1989) |
| + generalization training | COGNITIVE<br>BEHAVIORAL | Ellis et al. (1987a, 1987b)<br>Stokes & Baer (1977) |

Field testing for the curriculum involved special education teachers and students with a wide variety of learning problems. In different grouping arrangements, students with disabilities acquired computational skills, increased computational rates, generalized skills, and demonstrated independent use of a mnemonic strategy for solving difficult problems (Mercer & Miller, 1992c). The published manuals reflect empirical findings as well as teacher and student suggestions generated during field testing.

Effective in special education settings, the Strategic Math Series might offer mainstream teachers the "empirically validated" (Dixon & Carnine, 1990, p. 1) approach that would enable students with disabilities to succeed at mathematics in regular classroom settings. Would students with disabilities become proficient at solving computational facts (specifically multiplication) when the curriculum was used for large-group instruction under regular classroom conditions? How would the performance of mainstreamed students with disabilities compare with the performance of peers without disabilities receiving the same instruction under the same conditions? Answering those questions was the purpose of this investigation.

## Review of Methods

Subjects for this investigation were 13 students with disabilities (exceptional subjects) and 99 of their peers without disabilities in six second grade classrooms in a public school. Twelve exceptional subjects were labeled as having specific learning disabilities (SLD). One exceptional subject was classified as having an emotional disability (EH). All exceptional subjects were mainstreamed into regular second grade classes for instruction in mathematics. Subjects without disabilities represented typical ability and skill levels found in mainstream elementary classrooms with the exception of students who were excluded for low scores on tool and prerequisite skill screening. Multiplication Facts 0 to 81 (Mercer & Miller, 1992b) from the Strategic Math Series replaced the basal unit on multiplication. Regular classroom teachers conducted all instruction.

Two dependent variables were monitored throughout the investigation. The first dependent variable was subjects' untimed performance on 10 independent practice problems included with each lesson of the curriculum used for instruction. The second dependent variable was the number of correct and incorrect digits written per minute on a probe of basic multiplication facts. The probe was administered for several days prior to instruction as well as following each lesson of the curriculum.

The independent variable was defined as large-group instruction using Multiplication Facts 0 to 81 (Mercer & Miller, 1992b) from the Strategic Math Series. Groups ranged in size from 25-27 students. The 21 sequential lessons in the curriculum were divided into two instructional conditions. The first instructional condition emphasized understanding of the mathematical process through 10 lessons that followed the C-R-A sequence. A pretest preceded the first lesson, and a posttest followed lesson 10. The second instructional condition included 11 lessons. In the second condition, the instructional emphasis was on development of computational fluency and application of the skill to word problems.

Two studies were conducted. In each study, a single subject multiple baseline design was used to evaluate the effects of instruction and establish experimental control.

## Review of the Results and Limitations

Three experimental questions were asked about the effects of instruction on subjects' understanding of multiplication, computational fluency, and performance levels within a large group when classroom teachers followed the procedures outlined in Multiplication Facts

0 to 81 (Mercer & Miller, 1992b) from the Strategic Math Series for large-group instruction.

Question 1

The first experimental question investigated the effects on accuracy (in terms of criterion responding) for students with disabilities. The findings showed that exceptional subjects reached criterion levels of accurate responding and improved from pretest to posttest during large-group instruction.

During the first instructional condition, teachers presented an advance organizer, described and modeled the process, and conducted guided practice prior to students' independent practice with 10 problems on each lesson. Movement from one lesson to the next was controlled by a mastery criterion of 80% on the 10 independent practice problems. Students were required to repeat lessons with scores below 80%. For the most part, teachers handled the mastery requirement by circulating as students worked on independent practice problems. As students finished, teachers graded the problems and provided immediate feedback. If students scored below the 80% criterion, the lesson was immediately retaught. Within that same session, students corrected problems that were wrong, and the learning sheet was graded again. Across both studies, no exceptional subject scored below the 80% criterion on more than 2 lessons of the first instructional condition. All exceptional subjects reached the 80% criterion on their first attempt on at least 78% of the lessons, and median scores for the first instructional condition ranged from 80% to 100%.

In general, student absences were brief during the first instructional condition. When students returned to school, missed lessons were made up in sequence prior to introduction of a new lesson to the entire group. Teachers also reported that lessons using manipulative objects required substantially more time than that typically allotted for mathematics instruction.

The posttest contained 20 problems and followed lesson 10. Problems were not controlled for factor size, and students were required to demonstrate independent use of the mnemonic strategy taught during lesson 7 to reach a mastery criterion of 90%. Four exceptional subjects reached 90% on their first attempt. Seven exceptional subjects repeated lesson 10 and the posttest and reached 90% on their second attempt. One subject required an additional reteaching session, and 1 subject did not retake the posttest.

Examination of the posttests and scratch paper turned in with the posttests showed that, in general, subjects used the mnemonic strategy correctly but made counting mistakes when solving larger problems. For example, the second problem on the posttest, 9 X 9, required a drawing of nine lines with nine tallies on each line. To arrive at an answer, students could either count each tally or add successive groups of nine tallies. Wrong answers were typically in the high 70s (77, 78, 79) or low 80s (80, 82, 83, 84). Teachers commented that many students who relied on tally-by-tally counting made the drawings so small that they were unable to count the number of tallies accurately. Students who relied on adding successive groups of tallies were generally accurate until they reached the sixth group (54). Teachers also commented that typical classroom interruptions made it difficult for students to concentrate on counting accurately to higher numbers. For improving students' performance on the posttest, teachers suggested raising the mastery criterion to 90% on lessons 1-10. Teachers also recommended administering the posttest in two sessions to avoid fatigue for younger students.

During the second instructional condition, students learned an additional mnemonic strategy for word problems and began independent practice following the advance organizer. Independent practice included word problems. On several learning sheets, students were required to write and solve their own word problems. Students were not required to repeat lessons with scores below 80%. Across both studies, 12 exceptional subjects reached the 80% criterion on at least 33% of the lessons, and median scores for the second instructional condition ranged from 50% to 100%.

Numerous and longer absences due to viral illnesses occurred during the second instructional condition. Rather than waiting to have the entire group present, teachers continued instruction for students who were present. Typically, when students returned after long absences, they rejoined the group without making up all of the missed lessons. Group feedback was frequently used for typical mistakes made on solving and writing word problems. Teachers commented that reading, writing, and spelling difficulties sometimes resulted in incomplete word problems for exceptional subjects. The manual suggests modifying the requirements by reading problems to students or having students dictate their word problems. Teachers reported that it was difficult in large group situations to provide those modifications themselves and suggested using peers to work with students who needed more help.

Two other problems were encountered during the second instructional condition. Since repetition of the lesson was no longer required when students scored below 80%, teachers did not circulate as much during independent practice and collected papers at the end of the session to score and provide group feedback. Occasionally, papers were lost or turned in without names. Teachers suggested that, particularly for younger students, those problems might be avoided by including name lines on the learning sheets or by providing individual student workbooks.

### Question 2

The second experimental question investigated the effects on fluency (in terms of criterion responding) for students with disabilities. The results indicated that, while 12 of the 13 subjects failed to reach the criterion of 30 per minute for the number of correct digits, other effects were observed during within and between condition analyses.

Continuous monitoring prior to instruction (baseline) showed that overall performance did not improve for any exceptional subject on a timed probe of multiplication facts prior to instruction. For most subjects, the number of correct digits decreased or stayed the same while the number of incorrect digits increased during baseline.

During the first instructional condition, probes were administered after each lesson, but subjects received no feedback on their timed performance. Although performance was variable, 10 exceptional subjects increased the number of correct digits and decreased the number of incorrect digits as they progressed through the first 10 lessons.

During the second instructional condition, fluency practice and feedback became part of the instructional cycle. For 11 exceptional subjects, there were increases in the number of correct digits accompanied by decreases in the number of incorrect digits. Correct responding remained variable, but incorrect responding stabilized for most subjects. Although only 1 subject reached 30 correct digits, 10 subjects ended the investigation with correct responses still increasing. Only 4 subjects failed to reduce the number of incorrect digits to the criterion level of two or fewer per minute. Three of the 4 subjects were still reducing incorrect digits at the end of the investigation.

All of the exceptional subjects who failed to demonstrate crossover (more correct than incorrect responses) prior to instruction reached and maintained crossover during one of the two instructional cycles. Crossover appeared to be a highly individual phenomenon and occurred at different points among classrooms as well as among exceptional subjects within each classroom.

From the baseline phase to the end of instruction, overall percentages of change (based on median rates) ranged from increases of 71% to 1800% for the number of correct digits. The number of incorrect digits decreased by a range of 22% to 100%. Eleven subjects increased the number of correct digits by 150% or more, and 8 subjects reduced the number of incorrect digits by 94% or more.

Teachers commented that, although students were not very fast at the end of the investigation, they were confident and accurate in their use of the mnemonic strategy to solve unfamiliar problems. That observation was confirmed by the differences in variability noted for correct and incorrect responses. Five different versions of the probe were distributed throughout instruction, and students were directed to solve problems in order without skipping. As they encountered unfamiliar fact problems, their performance may have been affected by careful use of the strategy to arrive at a correct answer, particularly after repeating the posttest due to counting errors. Consequently, the number of correct digits varied considerably depending on the difficulty level of the problems, but variability decreased for incorrect digits as students became more consistently accurate when using the strategy.

Teachers also reported that a fatigue factor may have affected performance on the probes by the end of the investigation. By lesson 21, students had been exposed to 25 or more probes. According to teachers, students may have been tiring of the routine or may have been solving problems and writing answers as quickly as they could. Another complicating factor in building speed was the difficulty of scheduling instruction and feedback at the end of the year. During the last 2 weeks of instruction, word problems on the learning sheets for each lesson required so much teacher attention that individual monitoring and feedback on the probes was often replaced with group feedback. In addition, teachers occasionally required some assistance from other trained adults in scoring and recording student progress. Teachers commented that, in the future, they would begin using the curriculum earlier in the school year.

### Question 3

For the third experimental question, the performance of individual students with disabilities was evaluated in terms of the performance of normally achieving peers in the same group on measures of accuracy and fluency. Findings indicated that exceptional subjects frequently performed as well as their classmates without disabilities on both the independent practice problems and the timed probes. In addition, 12 exceptional subjects reached or exceeded the median score of classmates without disabilities on measures of both accuracy and fluency. One exceptional subject reached the group median score on the learning sheets but not on the timed probes.

For use of this curriculum as an effective mainstream intervention for students with disabilities, performance in the same range as their normally achieving classmates on 80% of the measures was set as the evaluative criterion. Across both studies, 11 exceptional subjects performed within range on more than 80% of the measures for both accuracy and fluency. The other 2 subjects reached the criterion on the timed probes but were below the criterion on the learning sheets (75% and 79%).

The distribution of both accuracy and fluency measures during separate instructional phases confirmed the similarity of performance for exceptional and normally achieving subjects. The only notable difference occurred for accuracy scores during the word problem phase of instruction (lessons 11-21). With the requirement for reading, writing, and solving word problems, a higher percentage of exceptional subjects failed to reach the 80% mastery criterion.

Teachers also observed that exceptional subjects performed as well as their classmates without disabilities during the first 10 lessons and on the timed probes. Differences became apparent with the reading and writing requirements of the word problems in the second instructional sequence. Beginning with lesson 12, exceptional subjects completed fewer learning sheets and required more help than their classmates. As mentioned previously, teachers remarked that peers could be helpful for reading problems aloud and writing dictated problems. Several teachers also suggested the formation of small groups that would continue to receive guided practice prior to independent practice.

### Limitations

A rural public school was the setting for this investigation, and findings should not be generalized to other settings without replication of results. In this investigation, exceptional subjects included 12 students with learning disabilities and 1 student with an emotional handicap who were mainstreamed into second grade classes for mathematics instruction. Caution should be exercised in extending the findings to students in other grade levels or to students with other or more severe disabilities. The curriculum materials used in this investigation replaced an introductory unit on multiplication. Extrapolation of results should not be made to other mathematical skills or to more advanced levels of multiplication without further investigation.

An additional limitation pertains to the curriculum itself. For students in second grade, the fluency criterion of 30 correct digits may be too high for developmental levels of motor skills. Out of 99 normally achieving students, 91 students had median levels below 20 correct digits per minute during fluency practice. All of the 13 exceptional subjects and the 11 excluded students were below 20 correct digits per minute during fluency practice.

## Implications and Suggestions for Future Research

The results of this investigation allowed the investigator to conclude that Multiplication Facts 0 to 81 (Mercer & Miller, 1992b) from the Strategic Math Series is appropriate for large-group instruction that includes mainstreamed students with disabilities. Several implications and future research directions are suggested by the findings in this investigation.

## Implications

The findings of this investigation clearly have implications for second grade teachers who are expected to introduce multiplication to large groups of students with varying ability and skill levels. Implications are also warranted about the importance of curriculum materials in facilitating effective teaching.

### Implications for teachers.

When teachers follow the scripted procedures in the Multiplication Facts 0 to 81 (Mercer & Miller, 1992b) manual from the Strategic Math Series, the instructional climate is predictable and consistent. Pretest information obtained prior to instruction assists teachers in selecting appropriate content and gaining student commitment. Lessons occur within an instructional cycle that includes explicit teaching, practice, monitoring, feedback, and generalization.

Following the presentation of new information in each lesson, students have the opportunity to practice the skill with teacher guidance. Independent practice follows guided practice. Practice opportunities include generalization of the skill to complex word problems. A specified mastery criterion controls movement within the curriculum and provides teachers with data based information for planning further instruction and delivering feedback. The data based information generated through frequent monitoring allows teachers to individualize instruction and ensure successful learning opportunities. As indicated by students' survey responses, successful learning opportunities generate positive attitudes toward multiplication.

As demonstrated in this investigation, students with learning problems can develop computational accuracy and fluency similar to that of their normally achieving classmates when instructional conditions are controlled through a structured curriculum. Very importantly, with mastery of the skill and the mnemonic strategy for independent problem solving, scores for students with learning problems will not be the lowest among their peers.

*Implications related to curriculum materials*. The results of this investigation have important implications for developers of curriculum materials. Traditionally, curriculum development has occurred within specific, and often rigid, theoretical orientations. Few materials exist that successfully integrate theories and empirical evidence across psychological and educational paradigms.

The curriculum used in this investigation clearly reflects the influence of several psychological and educational orientations. Development of the *Strategic Math*

Series was not confined within specific paradigms. Instead, developers of the series integrated theoretical tenets across diverse views of the teaching-learning process (Mercer & Miller, 1992c). The series was field tested for students with a variety of learning problems in special education settings. In this investigation, the curriculum materials were used with students with similar learning problems but in a regular classroom setting. The effectiveness of the curriculum as a mainstream intervention contradicts traditional impressions about the need for adherence to a particular theoretical paradigm.

A multitheoretical approach to instruction that utilizes the best available knowledge across theoretical paradigms may improve education in general and close the gap that currently exists between regular and special education. Collaboration and cooperation may become more of a reality when materials are available that blend ingredients familiar to teachers trained in different traditions. Development of multitheoretically based, empirically tested curriculum materials is needed at all grade levels and in all instructional disciplines.

This investigation also has implications for those responsible for curriculum planning and selection of materials. Even with the support of a rich theoretical foundation, developers of the Strategic Math Series carefully field tested the materials with a variety of students and different mathematical skills. The published manuals reflect empirical findings as well as teacher and student suggestions generated during field testing. The refinements that occurred during field testing contributed to the success of the intervention with larger groups of students under less controlled classroom conditions. When selecting materials, curriculum planners should examine field testing information critically to evaluate the appropriateness of the content and procedures for target students. Field testing should be an integral part of curriculum development, and materials that have not been field tested should be used cautiously.

### Future Research

In this investigation, Multiplication Facts 0 to 81 (Mercer & Miller, 1992b) from the Strategic Math Series was found to be an effective mainstream intervention for large-group instruction at the elementary level. Explicit, scripted instruction using a concrete to representational to abstract sequence that included a mnemonic solution strategy proved to be manageable and effective for regular classroom use. Multitheoretical in design, the materials of the Strategic Math Series offer a rich foundation for evaluating the effects of various theoretical components.

Additional information would support future development of curriculum materials with a similar multitheoretical foundation.

From a cognitive viewpoint, future research should identify the strategies developed by students on their own without explicit instruction in the mnemonic strategy DRAW. Differences among strategies developed independently by students with various learning characteristics should also be investigated.

With respect to the behavioral components, future research should investigate the effects of using reinforcement systematically during fluency practice. The effects of instruction on prerequisite and related skills should also be examined.

Curriculum designers should investigate the level of scripting necessary for various instructional procedures. In addition, knowledge about the effective use of large-group feedback and correction procedures would be beneficial for classroom teachers.

Since this investigation is the first use of the Strategic Math Series with very large groups of students, additional investigative efforts might focus on curriculum modifications in response to teacher and student comments. Specifically, procedures used for setting and communicating mastery criteria as well as monitoring progress and delivering feedback might be made more efficient for regular classroom use without losing the effectiveness demonstrated in this investigation.

On untimed independent practice problems, the effects of requiring mastery for all 21 lessons should be investigated as well as the effects of raising the mastery criterion to 90% during the first 10 lessons. Regarding the fluency criterion of 30 correct digits per minute, modification might be necessary for younger students. Since only 6.5% of the second grade students in this investigation reached a median level of 20 correct digits per minute during fluency practice, retention of the skill should be tested for 15 and 20 digits per minute with students of various ages and levels of motor skills.

Modifications to improve students' rates of performance might include separating fluency practice and instruction in complex word problems. As students worked toward a specified fluency criterion, they would continue to solve the type of simple word problems included in the first 10 lessons. After reaching the fluency criterion, the FAST DRAW strategy and more complex word problems would be introduced (lessons 11-21). The effects of completing the C-R-A sequence and fluency practice in second grade and requiring mastery of word problems during third grade should also be investigated. To allow more teacher time for checking, recording, and communicating individual progress on the timed probes, future investigations should not include instruction during the final week of the school year. Concerning communication of performance goals and expectations, would a verbal contract be as effective as a written contract? What levels of fluency would students reach if a separate commitment for mastery was made during fluency practice?

In the manuals of the Strategic Math Series, individual feedback is based on continuous monitoring of performance on every lesson. Would performance deteriorate if, after successful performance at the concrete level, monitoring and feedback occurred across every three lessons? If individual monitoring and feedback were maintained through the first 10 lessons (C-R-A sequence), could students work in pairs or small teams during the final 11 lessons (word problems and fluency practice)?

It would be valuable to know if returning to the describe, model, and guided practice cycle would improve the performance of students who have difficulty with the word problems found in lessons 11-21. The use of the multiplication manual should also be investigated with older students who have been introduced to the skill with traditional curriculum materials but are having difficulty solving fact problems independently.

The effectiveness of other manuals in the Strategic Math Series should be investigated under regular classroom conditions. The effectiveness of this curriculum should also be compared with other empirically validated scripted series. Finally, the unexpected finding that students with low scores in addition learned to solve multiplication problems accurately merits further investigation.

# APPENDIX A
# SCREENING INSTRUMENT AND SCRIPTED DIRECTIONS

# SCREENING INSTRUMENT

**NAME** ______________________________

**DATE** ______________________________

**TEACHER** ______________________________

| | |
|---|---|
| # DIGITS/1 MIN. | |
| MISSING NUMBERS | % |
| ADDITION FACTS | % |

**WRITE DIGITS (PAGE 1) ---------------------->**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
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| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |

**FILL IN THE MISSING NUMBERS (PAGE 2)**

1 2 3 ___ 5 6 ___ 8 9

10 11 12 ___ 14 15 16 17 ___

___ 20 21 ___ 23 24 25 26 27

28 29 30 31 ___ 33 34 ___ 36

37 38 39 ___ 41 42 43 ___ 45

___ 47 48 49 50 ___ ___ 53 54

55 56 ___ 58 59 ___ 61 62 ___

64 65 66 67 68 ___ 70 71 72

73 74 75 76 ___ 78 79 ___ ___

**WRITE THE ANSWERS (PAGE 3)**

$$\begin{array}{rrrrr} 5 & 7 & 4 & 8 & 3 \\ +\ 4 & +\ 3 & +\ 2 & +\ 7 & +\ 6 \\ \hline \end{array}$$

$$\begin{array}{rrrrr} 8 & 7 & 6 & 3 & 9 \\ +\ 3 & +\ 7 & +\ 0 & +\ 4 & +\ 8 \\ \hline \end{array}$$

$$\begin{array}{rrrrr} 7 & 9 & 8 & 9 & 8 \\ +\ 5 & +\ 2 & +\ 5 & +\ 9 & +\ 2 \\ \hline \end{array}$$

$$\begin{array}{rrrrr} 4 & 6 & 4 & 5 & 7 \\ +\ 9 & +\ 6 & +\ 8 & +\ 3 & +\ 6 \\ \hline \end{array}$$

# SCRIPTED DIRECTIONS FOR SCREENING INSTRUMENT

---

## DIRECTIONS FOR PAGE 1

You will need: screening instrument, pencils, stopwatch or clock with second hand.

Hand out pencils and the three-page screening instrument. When all students are ready, say:

"This is a review of writing numbers, counting, and adding numbers. Please look at page 1. Find the line with the word NAME. Write your first and last names on that line. Now, find the line with the word DATE. Write today's date like this (SHOW DATE ON BOARD). Next to TEACHER, write my name (WRITE YOUR NAME ON BOARD).

Now look at the boxes on page 1. This part of the review is to see how well you can write the numbers from 0 to 9. When I say BEGIN, write the numbers from 0 to 9 in the boxes. The arrow (---->) tells you to go across. One number goes in each box. If you finish a row before I say STOP, go on to the next row and start the numbers again. Work quickly and neatly. You will have 1 MINUTE to write the numbers from 0 to 9 as many times as you can.

What numbers will you be writing?
(WAIT FOR RESPONSE **0-9** FROM ALL STUDENTS)

Where will you write the numbers?
(WAIT FOR RESPONSE **IN THE BOXES** FROM ALL STUDENTS)

How many numbers will you write in each box?
(WAIT FOR RESPONSE **1** FROM ALL STUDENTS)

In which direction will you write the numbers?
(WAIT FOR RESPONSE **ACROSS** FROM ALL STUDENTS)

What should you do when you finish a row?
(WAIT FOR THE RESPONSE **GO ON TO THE NEXT ROW AND WRITE THE NUMBERS AGAIN** FROM ALL STUDENTS)

What word will you listen for to start writing?
WAIT FOR THE RESPONSE **BEGIN** FROM ALL STUDENTS)

How long will you have?
(WAIT FOR THE RESPONSE **1 MINUTE** FROM ALL STUDENTS)

What word will you listen for to stop writing?
(WAIT FOR THE RESPONSE **STOP** FROM ALL STUDENTS)

Good listening! Pencils ready? Please BEGIN."

After 1 minute, say "STOP" and ask students to turn to page 2.

## DIRECTIONS FOR PAGES 2 AND 3

When all students are looking at page 2, say:

"The part of the review on page 2 is to see how well you can count. The numbers on this page begin with 1 and go past 79. Some numbers are missing. Everywhere you see a line, a number is missing. You are to write in the numbers that are missing. You will not be timed. You will have as long as you need.

When you finish page 2, please go on to page 3. On page 3, you will show how well you can add numbers. Everyone look at page 3 now. There are 20 addition problems on the page. You are to write the answers to the problems. Please try all of the problems on the page.

Let's go back now to page 2.
What are you supposed to do on page 2?
(WAIT FOR THE RESPONSE **WRITE THE MISSING NUMBERS** FROM ALL STUDENTS)

How much time do you have to write the numbers?
(WAIT FOR THE RESPONSE **AS MUCH TIME AS WE NEED** FROM ALL STUDENTS)

What should you do when you finish page 2?
(WAIT FOR THE RESPONSE **GO ON TO PAGE 3** FROM ALL STUDENTS)

What kind of problems are on page 3?
(WAIT FOR THE RESPONSE **ADDITION** FROM ALL STUDENTS)

Good listening again. Now, please find the first missing number on page 2. Write the number that should be in that spot. Do the same for the rest of this page. Then go on to page 3. Raise your hand when you are finished, and I will come to your desk.”

## APPENDIX B
## SUMMARY OF SCREENING RESULTS ACROSS SIX CLASSROOMS

| | DIGITS/MINUTE | | COUNTING TO 81 | | BASIC + PROBLEMS | |
|---|---|---|---|---|---|---|
| Classroom 1 | range | median | range | median | range | median |
| Group N=16 | 30-60 | 47.5 | 90%-100% | 100% | 95%-100% | 100% |
| S 1.1 | 41 | | 100% | | 100% | |
| S 1.2 | 42 | | 100% | | 100% | |
| S 1.3 | 38 | | 95% | | 100% | |
| Classroom 2 | range | median | range | median | range | median |
| Group N=17 | 47-84 | 60 | 95%-100% | 100% | 85%-100% | 100% |
| S 2.1 | 40 | | 95% | | 95% | |
| S 2.2 | 43 | | 100% | | 75% | |
| Classroom 3 | range | median | range | median | range | median |
| Group N=19 | 30-70 | 50 | 90%-100% | 100% | 80%-100% | 100% |
| 3.1 | 51 | | 100% | | 100% | |
| Classroom 4 | range | median | range | median | range | median |
| Group N=16 | 30-100 | 53.5 | 95%-100% | 100% | 90%-100% | 100% |
| 4.1 | 56 | | 100% | | 95% | |
| 4.2 | 50 | | 100% | | 95% | |
| 4.3 | 64 | | 100% | | 95% | |

| | DIGITS/MINUTE | | COUNTING TO 81 | | BASIC + PROBLEMS | |
|---|---|---|---|---|---|---|
| Classroom 5 | range | median | range | median | range | median |
| Group N=18 | 38-88 | 51.5 | 85%-100% | 100% | 95%-100% | 100% |
| 5.1 | 80 | | 100% | | 100% | |
| Classroom 6 | range | median | range | median | range | median |
| Group N=13 | 32-100 | 62 | 95%-100% | 100% | 75%-100% | 95% |
| 6.1 | 43 | | 100% | | 100% | |
| 6.2 | 80 | | 100% | | 100% | |
| 6.3 | 50 | | 100% | | 95% | |

# APPENDIX C
# SAMPLES OF STUDENT MATERIALS FROM MULTIPLICATION FACTS 0 TO 81

Note: Material from Multiplication Facts 0 to 81 by C. D. Mercer and S. P. Miller, 1992, Lawrence, KS: Edge Enterprises. Copyright 1992 by C. D. Mercer and S. P. Miller. Reprinted by permission.

# Pretest

| | | | |
|---|---|---|---|
| 1<br>x 0 | 7<br>x 1 | 8<br>x 7 | 8<br>x 6 |
| 8<br>x 5 | 6<br>x 6 | 2<br>x 2 | 7<br>x 9 |
| 7<br>x 6 | 9<br>x 2 | 8<br>x 8 | 3<br>x 7 |
| 3<br>x 5 | 3<br>x 8 | 4<br>x 6 | 6<br>x 9 |
| 0<br>x 3 | 9<br>x 9 | 3<br>x 4 | 9<br>x 3 |

# Learning Contract

I, ____________________, agree to learn Multiplication Facts 0 to 81. If I work hard, I will learn to solve multiplication facts quickly and accurately. This will help me understand math and get better grades.

_________________________ _______________
**Student Signature** **Date**

I, ____________________, agree to do whatever I can to help you learn how to multiply. I will follow the instructions outlined in the book, Multiplication Facts 0 to 81, and I will put all my creative energies into showing you how multiplication is a valuable skill to learn.

_________________________ _______________
**Teacher Signature** **Date**

# Learning Sheet 1

## Describe and Model

1) 4
x 3
______

2) 5 x 1 = ______

3) 3
x 2
______

## Guided Practice

4) 5
x 3
______

5) 3 x 3 = ______

6) 1
x 3
______

## Independent Practice

7) 5
x 1
______

8) 2 x 2 = ______

9) 2
x 6
______

10) 3
x 7
______

11) 4 x 5 = ______

12) 1
x 4
______

## Problem Solving

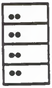
13) 7 groups
of 3
______ ____________
____________

14) 2 groups
of 5
______ ____________
____________

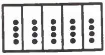
# Learning Sheet 4

## Describe and Model

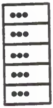
1) 4
x 2



2) 5 x 4 = ____

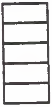


3) 5
x 3

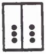


## Guided Practice

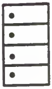
4) 5
x 0



5) 2 x 3 = ____

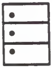


6) 4
x 1

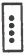


## Independent Practice

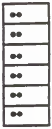
7) 3
x 1



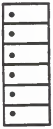
8) 1 x 4 = ____



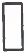
9) 6
x 2



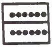
10) 6
x 1



11) 1 x 0 = ____

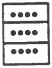


12) 2
x 6

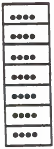


## Problem Solving: "Dots"

13) 3 groups
of 4 ____
____



14) 7 groups
of 4 ____
____



## Learning Sheet 10

Describe and Model

**1)** 8
x 0

Guided Practice

**2)** 1
x 1

**3)** 2 x 2 = ____

**4)** 2
x 5

Independent Practice

**5)** 7
x 5

**6)** 7 x 7 = ____

**7)** 2
x 4

**8)** 1
x 0

**9)** 3 x 3 = ____

**10)** 1
x 4

Problem Solving

**11)** Mark has 4 trains.
Each train has 6 cars.
There are cars in all.

**12)** Jennifer has 5 homework papers.
Each paper has 2 word problems.
There are word problems in all.

**13)** Al played 6 football games.
He scored 4 touchdowns in each game.
He scored touchdowns altogether.

# Posttest

| | | | |
|---|---|---|---|
| 0<br>x 3 | 9<br>x 9 | 3<br>x 4 | 9<br>x 3 |
| 3<br>x 5 | 3<br>x 8 | 4<br>x 6 | 6<br>x 9 |
| 7<br>x 6 | 9<br>x 2 | 8<br>x 8 | 3<br>x 7 |
| 8<br>x 5 | 6<br>x 6 | 2<br>x 2 | 7<br>x 9 |
| 1<br>x 0 | 7<br>x 1 | 8<br>x 7 | 8<br>x 6 |

# Learning Sheet 18

Independent Practice

1) 1
x 7

2) 9 x 0 = _____

3) 1
x 3

4) 8
x 9

5) 9
x 8

6) 8
x 7

7) 7
x 8

Problem Solving

8) Doug, Alice, and Kevin bought new wheels for their skateboards. Each skateboard had 4 wheels. How many wheels did they buy altogether?

9) Mr. Greentree planted 7 rows of popcorn and 1 row of roses. There were 6 popcorn plants in each row. How many popcorn plants were in his garden altogether?

10)

Pig Game Practice

11) 9
x 9

12) 7
x 9

13) 8
x 8

14) 1
x 0

15) 0
x 0

16) 7
x 4

17) 8
x 5

18) 9
x 6

19) 6
x 7

20) 4
x 7

# Multiplication Minute

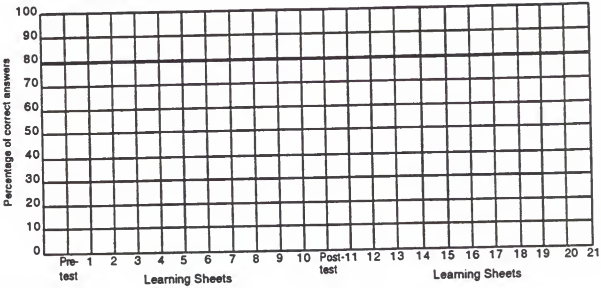
| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4<br>x 3 | 3<br>x 2 | 6<br>x 5 | 7<br>x 6 | 8<br>x 7 | 2<br>x 1 | 6<br>x 0 | 8<br>x 8 | 4<br>x 9 |
| 3<br>x 6 | 6<br>x 8 | 1<br>x 5 | 2<br>x 2 | 7<br>x 1 | 8<br>x 4 | 4<br>x 5 | 6<br>x 4 | 2<br>x 3 |
| 3<br>x 4 | 6<br>x 6 | 7<br>x 2 | 9<br>x 1 | 8<br>x 7 | 4<br>x 8 | 3<br>x 9 | 7<br>x 3 | 7<br>x 5 |
| 5<br>x 5 | 8<br>x 2 | 9<br>x 7 | 8<br>x 6 | 2<br>x 7 | 3<br>x 8 | 9<br>x 9 | 4<br>x 1 | 4<br>x 6 |
| 5<br>x 8 | 3<br>x 3 | 4<br>x 2 | 9<br>x 6 | 7<br>x 8 | 2<br>x 5 | 2<br>x 9 | 2<br>x 4 | 8<br>x 0 |
| 8<br>x 9 | 6<br>x 1 | 9<br>x 4 | 8<br>x 3 | 6<br>x 9 | 9<br>x 5 | 9<br>x 8 | 7<br>x 7 | 5<br>x 6 |

Multiplication Facts 0 to 81

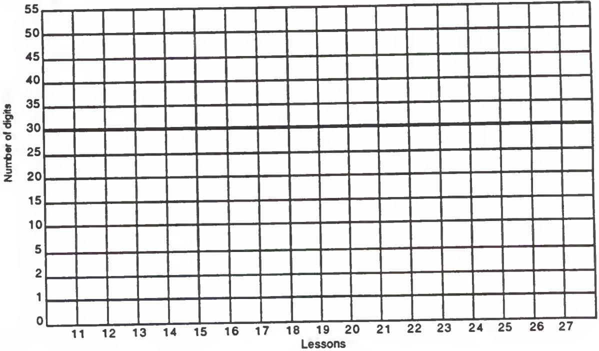
# MULTIPLICATION PROGRESS CHART

Name ______________________________

## Percentage Graph



## Rate Graph



| Rate Graph Key | |
|---|---|
| *Multiplication Minute* | *Facts Review* |
| ● # correct digits | ■ # correct digits |
| ○ # incorrect digits | □ # incorrect digits |

# APPENDIX D
# ALTERNATE FORMS OF THE MULTIPLICATION MINUTE

# Multiplication Minute ▲

Name ______________________________ Date _____________

Teacher _____________________ Lesson Number __________

Timing Period _____

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 3<br>X 5 | 4<br>X 1 | 2<br>X 7 | 3<br>X 6 | 4<br>X 2 | 5<br>X 9 | 5<br>X 3 | 1<br>X 8 | 1<br>X 2 |
| 7<br>X 3 | 3<br>X 6 | 8<br>X 4 | 3<br>X 7 | 2<br>X 9 | 3<br>X 9 | 6<br>X 0 | 6<br>X 2 | 9<br>X 5 |
| 7<br>X 6 | 7<br>X 8 | 6<br>X 9 | 8<br>X 5 | 5<br>X 1 | 6<br>X 3 | 1<br>X 7 | 4<br>X 6 | 2<br>X 8 |
| 9<br>X 6 | 8<br>X 2 | 9<br>X 7 | 2<br>X 3 | 8<br>X 7 | 2<br>X 2 | 3<br>X 8 | 6<br>X 5 | 1<br>X 4 |
| 5<br>X 4 | 4<br>X 3 | 8<br>X 9 | 4<br>X 0 | 3<br>X 0 | 5<br>X 2 | 4<br>X 9 | 3<br>X 2 | 7<br>X 2 |
| 9<br>X 8 | 7<br>X 9 | 7<br>X 1 | 5<br>X 7 | 1<br>X 1 | 4<br>X 8 | 9<br>X 4 | 9<br>X 0 | 4<br>X 7 |

# Multiplication Minute ☐

Name ______________________________ Date ____________

Teacher ______________________ Lesson Number __________

Timing Period _____

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 2<br>X 6 | 1<br>X 9 | 7<br>X 0 | 4<br>X 5 | 3<br>X 6 | 2<br>X 4 | 4<br>X 9 | 6<br>X 2 | 1<br>X 3 |
| 8<br>X 2 | 5<br>X 9 | 9<br>X 5 | 6<br>X 3 | 9<br>X 6 | 6<br>X 4 | 9<br>X 7 | 8<br>X 8 | 5<br>X 2 |
| 4<br>X 6 | 2<br>X 9 | 9<br>X 2 | 1<br>X 8 | 9<br>X 0 | 2<br>X 8 | 1<br>X 6 | 8<br>X 9 | 7<br>X 3 |
| 9<br>X 9 | 3<br>X 4 | 9<br>X 4 | 3<br>X 9 | 5<br>X 4 | 3<br>X 3 | 1<br>X 0 | 4<br>X 2 | 5<br>X 1 |
| 1<br>X 1 | 9<br>X 8 | 9<br>X 3 | 1<br>X 5 | 5<br>X 8 | 8<br>X 1 | 2<br>X 5 | 9<br>X 1 | 7<br>X 9 |
| 8<br>X 7 | 2<br>X 0 | 7<br>X 2 | 3<br>X 2 | 7<br>X 8 | 4<br>X 0 | 6<br>X 9 | 3<br>X 0 | 8<br>X 5 |

## Multiplication Minute *

Name ______________________ Date __________

Teacher ________________ Lesson Number ________

Timing Period _____

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 3 | 6 | 7 | 8 | 2 | 6 | 8 | 4 |
| X 3 | X 2 | X 5 | X 6 | X 7 | X 1 | X 0 | X 8 | X 9 |
| 3 | 6 | 1 | 2 | 7 | 8 | 4 | 6 | 2 |
| X 6 | X 8 | X 5 | X 2 | X 1 | X 4 | X 5 | X 4 | X 3 |
| 3 | 6 | 7 | 9 | 8 | 4 | 3 | 7 | 7 |
| X 4 | X 6 | X 2 | X 1 | X 7 | X 8 | X 9 | X 3 | X 5 |
| 5 | 8 | 9 | 8 | 2 | 3 | 9 | 4 | 4 |
| X 5 | X 2 | X 7 | X 6 | X 7 | X 8 | X 9 | X 1 | X 6 |
| 5 | 3 | 4 | 9 | 7 | 2 | 2 | 2 | 8 |
| X 8 | X 3 | X 2 | X 6 | X 8 | X 5 | X 9 | X 4 | X 0 |
| 8 | 6 | 9 | 8 | 6 | 9 | 9 | 7 | 5 |
| X 9 | X 1 | X 4 | X 3 | X 9 | X 5 | X 8 | X 7 | X 6 |

# Multiplication Minute ◆

Name ______________________ Date ____________

Teacher ________________ Lesson Number _________

Timing Period _____

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4<br>X 6 | 3<br>X 9 | 1<br>X 9 | 6<br>X 2 | 2<br>X 0 | 2<br>X 7 | 8<br>X 0 | 1<br>X 8 | 8<br>X 9 |
| 6<br>X 0 | 3<br>X 5 | 7<br>X 9 | 7<br>X 5 | 4<br>X 9 | 2<br>X 4 | 7<br>X 8 | 2<br>X 2 | 6<br>X 6 |
| 9<br>X 6 | 1<br>X 5 | 8<br>X 6 | 9<br>X 1 | 7<br>X 3 | 0<br>X 4 | 2<br>X 9 | 4<br>X 2 | 9<br>X 9 |
| 6<br>X 4 | 1<br>X 1 | 4<br>X 8 | 2<br>X 0 | 7<br>X 2 | 2<br>X 8 | 7<br>X 4 | 6<br>X 7 | 4<br>X 5 |
| 4<br>X 3 | 1<br>X 4 | 1<br>X 3 | 3<br>X 4 | 3<br>X 8 | 6<br>X 3 | 4<br>X 1 | 4<br>X 4 | 5<br>X 8 |
| 5<br>X 5 | 4<br>X 7 | 8<br>X 4 | 1<br>X 2 | 7<br>X 7 | 8<br>X 8 | 1<br>X 7 | 8<br>X 3 | 9<br>X 4 |

# Multiplication Minute ◆

Name ______________________ Date __________

Teacher ________________ Lesson Number ________

Timing Period _____

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 3<br>X 8 | 6<br>X 6 | 6<br>X 0 | 5<br>X 2 | 3<br>X 3 | 4<br>X 9 | 5<br>X 6 | 1<br>X 5 | 8<br>X 1 |
| 1<br>X 9 | 5<br>X 8 | 3<br>X 0 | 9<br>X 6 | 8<br>X 6 | 2<br>X 3 | 1<br>X 8 | 7<br>X 9 | 6<br>X 7 |
| 1<br>X 7 | 1<br>X 4 | 9<br>X 7 | 1<br>X 0 | 7<br>X 8 | 0<br>X 7 | 2<br>X 4 | 2<br>X 0 | 7<br>X 6 |
| 7<br>X 3 | 9<br>X 5 | 8<br>X 3 | 3<br>X 9 | 2<br>X 5 | 4<br>X 0 | 2<br>X 1 | 6<br>X 0 | 5<br>X 4 |
| 8<br>X 7 | 6<br>X 2 | 6<br>X 8 | 7<br>X 0 | 9<br>X 1 | 7<br>X 5 | 6<br>X 3 | 7<br>X 1 | 7<br>X 7 |
| 2<br>X 2 | 7<br>X 4 | 8<br>X 9 | 3<br>X 5 | 0<br>X 2 | 6<br>X 5 | 8<br>X 2 | 2<br>X 7 | 4<br>X 2 |

# APPENDIX E
# TEACHER SURVEY

## Multiplication Facts 0 to 81

Please circle the number for your response to each question.

1=Excellent 2=Good 3=Fair 4=Poor 5=Unacceptable

1. What did you think of the teaching script for . . .

| | | | | | |
|---|---|---|---|---|---|
| computation problems? | 1 | 2 | 3 | 4 | 5 |
| word problems? | 1 | 2 | 3 | 4 | 5 |
| the **D-R-A-W** strategy? | 1 | 2 | 3 | 4 | 5 |
| the **F-A-S-T D-R-A-W** strategy? | 1 | 2 | 3 | 4 | 5 |
| feedback? | 1 | 2 | 3 | 4 | 5 |
| fluency practice ? | 1 | 2 | 3 | 4 | 5 |

2. How would you describe . . .

| | | | | | |
|---|---|---|---|---|---|
| the concrete lessons? | 1 | 2 | 3 | 4 | 5 |
| the representational lessons? | 1 | 2 | 3 | 4 | 5 |
| the **D-R-A-W** strategy? | 1 | 2 | 3 | 4 | 5 |
| the **F-A-S-T D-R-A-W** strategy? | 1 | 2 | 3 | 4 | 5 |
| the fluency practice? | 1 | 2 | 3 | 4 | 5 |
| the word problem sequence? | 1 | 2 | 3 | 4 | 5 |
| the learning sheets? | 1 | 2 | 3 | 4 | 5 |
| the use of scripts? | 1 | 2 | 3 | 4 | 5 |
| the lesson sequence (advanced organizer, etc.)? | 1 | 2 | 3 | 4 | 5 |
| timings for lessons 11-21? | 1 | 2 | 3 | 4 | 5 |
| the use of individual progress charts? | 1 | 2 | 3 | 4 | 5 |
| specified criteria for mastery? | 1 | 2 | 3 | 4 | 5 |
| individual feedback? | 1 | 2 | 3 | 4 | 5 |
| the contract to learn? | 1 | 2 | 3 | 4 | 5 |
| the pretest? | 1 | 2 | 3 | 4 | 5 |
| the posttest? | 1 | 2 | 3 | 4 | 5 |
| the pig games? | 1 | 2 | 3 | 4 | 5 |

3. How accurate were pretest scores? 1 2 3 4 5

4. How accurate were posttest scores? 1 2 3 4 5

5. How would you describe your students' mastery of . . .

   multiplication facts? 1 2 3 4 5
   multiplication rules? 1 2 3 4 5
   word problems? 1 2 3 4 5
   the **D-R-A-W** strategy? 1 2 3 4 5

6. How did you like this math program? 1 2 3 4 5

7. How did your students like this math program? 1 2 3 4 5

8. Do you think this program is appropriate to use in regular education classrooms?

9. What did you like best about this math program?

10. What did you like least about this math program?

11. How would you compare this math program to other math programs that you have used?

12. Should student workbooks be provided?

13. Will you use this program again? If so, when in the school year would you begin?

14. Please describe any modifications that you found to be helpful or would like to see included in this program.

# APPENDIX F
# SUMMARY OF SURVEY RESPONSES FOR SIX TEACHERS

| Curriculum Component | Number of Surveys With Rating of 1 (Excellent) or 2 (Good) |
|---|---|
| Teaching Scripts for: | |
| • computation problems | 6/6 |
| • word problems | 6/6 |
| • the DRAW strategy | 6/6 |
| • the FAST DRAW strategy | 6/6 |
| • feedback | 6/6 |
| • fluency practice | 5/6 |
| Procedures and Materials | |
| • concrete lessons | 6/6 |
| • representational lessons | 6/6 |
| • DRAW | 6/6 |
| • FAST DRAW | 6/6 |
| • fluency practice | 4/6 |
| • word problem sequence | 6/6 |
| • learning sheets | 6/6 |
| • using scripts | 6/6 |
| • lesson sequence | 6/6 |
| • timings (lessons 11-21) | 5/6 |
| • use of progress charts | 5/6 |
| • mastery criteria | 4/5 |
| • individual feedback | 6/6 |
| • learning contract | 4/6 |
| • pretest | 5/6 |
| • posttest | 6/6 |
| • pig games | 6/6 |
| Accuracy of: | |
| • pretest scores | 6/6 |
| • posttest scores | 6/6 |
| Students' Mastery of: | |
| • multiplication facts | 5/6 |
| • multiplication rules | 6/6 |
| • word problems | 4/6 |
| • DRAW | 5/6 |

# APPENDIX G
# STUDENT SURVEY

## Multiplication Facts 0 to 81

Name ______________________ Date ______________

Please circle your answer for each question.

☺ ☹

I liked it. I didn't like it.

1. How did you like learning multiplication?

☺ ☹

2. How did you like using objects to figure out answers to problems?

☺ ☹

3. How did you like using drawings to figure out problems?

☺ ☹

4. How did you like the **D-R-A-W** strategy to solve problems by yourself?

☺ ☹

5. How did you like **F-A-S-T D-R-A-W** for word problems?

☺ ☹

6. How did you like learning to do problems quickly?

☺ ☹

7. How did you like the Multiplication Minute at the end of each lesson?

☺ ☹

8. How did you like seeing your scores on the progress chart?

9. How did you like it when your teacher gave you feedback on the problems that you did by yourself?

10. What was the best part of learning multiplication with *Multiplication Facts 0 to 81*?

11. What was the worst part of learning multiplication with *Multiplication Facts 0 to 81*?

12. Please write anything else that you would like to say about this math program.

# APPENDIX H
# PERCENTAGES OF POSITIVE SURVEY RESPONSES FOR NORMALLY ACHIEVING SUBJECTS, EXCEPTIONAL SUBJECTS, AND STUDENTS EXCLUDED DURING SCREENING OR BASELINE

| Curriculum Component | Normally Achieving N=97 | Exceptional N=11 | Excluded N=12 |
|---|---|---|---|
| • learning multiplication | 97% | 100% | 100% |
| • using objects | 80% | 100% | 92% |
| • using drawings | 76% | 100% | 92% |
| • DRAW | 85% | 91% | 92% |
| • FAST DRAW | 77% | 91% | 92% |
| • doing problems quickly | 96% | 100% | 100% |
| • _Multiplication Minute_ | 88% | 82% | 100% |
| • progress chart | 91% | 100% | 100% |
| • individual feedback | 97% | 100% | 92% |

# APPENDIX I
# TIME SAMPLING RECORDING FORM FOR PROCEDURAL RELIABILITY

# TIME SAMPLING RECORDING FORM FOR PROCEDURAL RELIABILITY

Teacher: ______________________ Observer: ______________________

| DATE ____________ | 30 | 60 | 30 | 60 | 30 | 60 | 30 | 60 | 30 | 60 | 30 | 60 | 30 | 60 | 30 | 60 | 30 | 60 | 30 | 60 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Give Advance Organizer | | | | | | | | | | | | | | | | | | | | |
| Describe or Model | | | | | | | | | | | | | | | | | | | | |
| Conduct Practice | | | | | | | | | | | | | | | | | | | | |
| Multiplication Minute/Facts Review | | | | | | | | | | | | | | | | | | | | |
| Give Feedback | | | | | | | | | | | | | | | | | | | | |

# APPENDIX J
# RAW DATA FOR EXCEPTIONAL SUBJECTS IN EACH CLASSROOM

RAW DATA FOR EXCEPTIONAL SUBJECTS IN CLASSROOM 1
(with group summary information)

| Subject | dig/min # | cntg % | add % | CTBS | B1 ●/X | B2 ●/X | B3 ●/X | B4 ●/X |
|---|---|---|---|---|---|---|---|---|
| 1.1 | 41 | 100 | 100 | 13 | 0/54 | 0/54 | 1/56 | 0/54 |
| 1.2 | 42 | 100 | 100 | 9 | 2/13 | 0/12 | 1/18 | 2/36 |
| 1.3 | 38 | 95 | 100 | N/A | 2/43 | 0/34 | 0/45 | 0/54 |
| Group Median | 47.5 | 100 | 100 | 69 | 3/32 | 2/22.5 | 2/19 | 3/36.5 |

| Subject | PRE % | C1 % ●/X | C2 % ●/X | C3 % ●/X | R4 % ●/X | R5 % ●/X | R6 % ●/X | MN7 ●/X | A8 % ●/X | A9 % ●/X | A10 % ●/X | POST % |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1.1 | 15 | 100<br>3/8 | 90<br>4/6 | 80<br>11/35 | 90<br>13/12 | 100<br>14/29 | 80<br>10/35 | 3/2 | 90<br>3/1 | 90<br>3/2 | 80<br>13/1 | 90 |
| 1.2 | 10 | 90<br>1/15 | 90<br>4/6 | 100<br>5/5 | 100<br>7/11 | 90<br>7/4 | 100<br>8/0 | 4/3 | 100<br>6/0 | 100 | 90<br>5/0 | 70<br>95 |
| 1.3 | 10 | 90<br>3/9 | 60 100<br>5/2 | 100<br>3/0 | 100<br>2/0 | 90<br>2/0 | 90<br>6/1 | 7/3 | 80<br>13/2 | 100<br>15/7 | 100<br>12/1 | 65 |
| Group Median | 12.5 | 90<br>3/7 | 90<br>4/2 | 90<br>8/1 | 100<br>5/2 | 90<br>7/1.5 | 100<br>9/2.5 | 6.5/1 | 90<br>10/1.5 | 100<br>14/2 | 90<br>16/0 | 80 |

| Subject | 11 % ●/X | 12 % ●/X | 13 % ●/X | 14 % ●/X | 15 % ●/X | 16 % ●/X | 17 % ●/X | 18 % ●/X | 19 % ●/X | 20 % ●/X | 21 % |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1.1 | 10/0 | 8/0 | 70<br>6/2 | 40<br>8/0 | 50<br>6/1 | 8/0 | 70<br>14/0 | 50<br>7/1 | 14/0 | | |
| 1.2 | 100<br>1/1 | 100<br>14/4 | 60<br>11/1 | 40<br>13/1 | 70<br>9/2 | 70<br>8/0 | 80<br>16/2 | 60<br>16/2 | 60<br>11/0 | 80 | 100 |
| 1.3 | 100<br>11/7 | 100<br>16/7 | 80<br>13/1 | 90<br>17/1 | 70<br>7/2 | 50<br>14/0 | 70<br>18/0 | 40<br>13/4 | 60<br>10/3 | 70<br>16/3 | |
| Group Median | 100<br>12/1 | 100<br>12/2 | 80<br>15/1 | 70<br>18/0 | 85<br>8/1 | 75<br>16/0 | 90<br>18/1 | 70<br>13/2 | 90<br>20/0 | 80<br>16/1 | 100 |

# RAW DATA FOR EXCEPTIONAL SUBJECTS IN CLASSROOM 2
(with group summary information)

| Subject | dig/min # | cntg % | add % | CTBS | B1 ●/X | B2 ●/X | B3 ●/X | B4 ●/X | B5 ●/X | B6 ●/X |
|---|---|---|---|---|---|---|---|---|---|---|
| 2.1 | 40 | 95 | 95 | 76 | 2/6 | 0/7 | 0/8 | 2/11 | 3/10 | 3/18 |
| 2.2 | 43 | 100 | 75 | 17 | 3/4 | 3/12 | 2/12 | 2/16 | 2/21 | 0/32 |
| Group Median | 60 | 100 | 100 | 88 | 4/2 | 5/3 | 5/4 | 6/6 | 4/7 | 4/10 |

| Subject | PRE % | C1 % ●/X | C2 % ●/X | C3 % ●/X | R4 % ●/X | R5 % ●/X | R6 % ●/X | MN7 ●/X | A8 % ●/X | A9 % ●/X | A10 % ●/X | POST % |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2.1 | 30 | 100<br>4/10 | 100<br>2/11 | 80<br>9/4 | 100<br>5/3 | 90<br>9/3 | 100<br>8/0 | 7/8 | 90<br>12/3 | 80<br>5/5 | 100<br>9/0 | 65<br>90 |
| 2.2 | 5 | 90<br>2/10 | 100<br>5/21 | 80<br>4/4 | 80<br>4/6 | 70 100<br>4/9 | 100<br>3/2 | 0/2 | 80<br>4/4 | 80<br>5/5 | 90<br>6/2 | 30<br>90 |
| Group Median | 15 | 100<br>4/7.5 | 100<br>8/2.5 | 90<br>8/3 | 100<br>6/14 | 100<br>8/9 | 90<br>8/2 | 5/1 | 90<br>10/4 | 100<br>7/3 | 100<br>9/1 | 75 |

| Subject | 11 % ●/X | 12 % ●/X | 13 % ●/X | 14 % ●/X | 15 % ●/X | 16 % ●/X | 17 % ●/X | 18 % ●/X | 19 % ●/X | 20 % ●/X | 21 % ●/X |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2.1 | 100<br>10/2 | 80<br>8/2 | 60<br>11/3 | 100<br>6/3 | 40<br>8/0 | 40<br>15/3 | 90<br>5/2 | 90<br>10/0 | 80<br>14/0 | 60<br>11/1 | 90<br>13/1 |
| 2.2 | 90<br>1/3 | 80<br>3/6 | 50<br>5/1 | 50<br>3/15 | | | 70<br>4/6 | 50<br>5/4 | 60<br>5/3 | 60<br>9/8 | 80<br>12/6 |
| Group Median | 90<br>12/1 | 100<br>10/1 | 95<br>13/0 | 80<br>6/2 | 75<br>14/0 | 90<br>16/0 | 90<br>6/2 | 70<br>10/1 | 90<br>16/1 | 95<br>12.5/1 | 90<br>15/0 |

# RAW DATA FOR EXCEPTIONAL SUBJECTS IN CLASSROOM 3
(with group summary information)

| Subject | dig/min<br>#| cntg<br>% | add<br>% | CTBS | B1<br>●/X | B2<br>●/X | B3<br>●/X | B4<br>●/X | B5<br>●/X | B6<br>●/X | B7<br>●/X | B8<br>●/X |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3.1 | 51 | 100 | 100 | 90 | 11/2 | 10/4 | 8/3 | 9/3 | 9/4 | 12/6 | 11/7 | 14/4 |
| Group Median | 50 | 100 | 100 | 69.5 | 4/5.5 | 4/8 | 3.5/12.5 | 4/12 | 3/14 | 6/27 | 5/19 | 7/14 |

| Subject | PRE<br>% | C1<br>%<br>●/X | C2<br>%<br>●/X | C3<br>%<br>●/X | R4<br>%<br>●/X | R5<br>%<br>●/X | R6<br>%<br>●/X | MN7<br>●/X | A8<br>%<br>●/X | A9<br>%<br>●/X | A10<br>%<br>●/X | POST<br>% |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3.1 | 50 | 70 100<br>7/3 | 100<br>14/2 | 80<br>12/3 | 100<br>11/4 | 100<br>17/7 | 100<br>16/2 | 6/7 | 100<br>11/1 | 100<br>9/1 | 100<br>19/1 | 80<br>100 |
| Group Median | 15 | 90<br>4.5/6 | 100<br>9/2 | 90<br>6/3 | 100<br>3.5/4.5 | 85<br>10/3.5 | 100<br>7.5/2 | 5/0 | 90<br>9.5/4.5 | 100<br>10/3 | 90<br>12/2 | 80 |

| Subject | 11<br>%<br>●/X | 12<br>%<br>●/X | 13<br>%<br>●/X | 14<br>%<br>●/X | 15<br>%<br>●/X | 16<br>%<br>●/X | 17<br>%<br>●/X | 18<br>%<br>●/X | 19<br>%<br>●/X | 20<br>%<br>●/X | 21<br>%<br>●/X |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 3.1 | 80<br>11/1 | 90<br>8/0 | 70<br>22/0 | 80<br>14/1 | 70<br>18/0 | 60<br>31/0 | 80<br>12/1 | 90<br>19/1 | 90<br>22/1 | 80<br>24/0 | 50<br>19/1 |
| Group Median | 90<br>12/0 | 100<br>8/2 | 80<br>14/0 | 70<br>7/2 | 70<br>14/0 | 70<br>17/0 | 80<br>10/2 | 70<br>16/2 | 80<br>18/1 | 90<br>12/2 | 90<br>14/1 |

RAW DATA FOR EXCEPTIONAL SUBJECTS IN CLASSROOM 4
(with group summary information)

| Subject | dig/min # | cntg % | add % | CTBS | B1 ●/X | B2 ●/X | B3 ●/X | B4 ●/X |
|---|---|---|---|---|---|---|---|---|
| 4.1 | 56 | 100 | 95 | 88 | 0/1 | 2/4 | 1/8 | 2/12 |
| 4.2 | 50 | 100 | 95 | 42 | 0/13 | 1/17 | 0/4 | 0/5 |
| 4.3 | 64 | 100 | 95 | N/A | 0/3 | 0/5 | | |
| Group Median | 53.5 | 100 | 100 | 65 | 2.5/6.5 | 4/10 | 2.5/9 | 3.5/13 |

| Subject | PRE % | C1 % ●/X | C2 % ●/X | C3 % ●/X | R4 % ●/X | R5 % ●/X | R6 % ●/X | MN7 ●/X | A8 % | A9 % ●/X | A10 % ●/X | POST % |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 4.1 | 15 | 40 100<br>2/5 | 90<br>4/7 | 60 100<br>9/3 | 100<br>8/10 | 90<br>13/2 | 90<br>8/4 | 9/4 | 100 | 80<br>8/7 | 90<br>10/5 | 100 |
| 4.2 | 20 | 90<br>3/12 | 90<br>5/17 | 100<br>6/15 | 100<br>14/15 | 90<br>8/7 | 90<br>13/36 | 19/51 | 90 | 100<br>12/27 | 100<br>21/39 | 100 |
| 4.3 | 15 | 70 90<br>3/2 | 80<br>2/6 | 80<br>4/17 | 90<br>9/13 | 80<br>3/18 | 90 | 5/0 | 100 | 90<br>10/15 | 90 | 95 |
| Group Median | 20 | 100<br>5/19 | 90<br>8/5 | 90<br>6/11 | 100<br>11/19 | 90<br>9/14 | 90<br>10/11.5 | 7/13 | 90 | 100<br>12.5/13.5 | 100<br>13/11.5 | 90 |

| Subject | 11 % ●/X | 12 % ●/X | 13 % ●/X | 14 % ●/X | 15 % ●/X | 16 % ●/X | 17 % ●/X | 18 % ●/X | 19 % ●/X | 20 % ●/X | 21 % |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 4.1 | 100<br>15/2 | 90<br>16/8 | 70<br>13/1 | 90<br>8/3 | 13/1 | 80<br>12/2 | 70 | | 70 | 80 | 80 |
| 4.2 | 100<br>8/7 | 100<br>16/20 | 70<br>26/14 | 70<br>10/5 | 100<br>19/5 | 100<br>24/5 | 100<br>13/6 | 14/10 | 90<br>21/10 | 18/8 | 100<br>19/7 |
| 4.3 | 100<br>2/0 | 100<br>1/1 | 70<br>9/0 | 60<br>5/0 | 100<br>14/0 | 40<br>14/0 | | | 80<br>16/2 | 80<br>12/0 | 80<br>22/0 |
| Group Median | 100<br>11/1 | 100<br>10/5 | 70<br>13/2 | 75<br>9.5/2 | 100<br>18/0 | 100<br>17/2 | 100<br>9/3.5 | 90<br>14.5/3 | 100<br>17/3.5 | 100<br>17.5/2 | 85<br>20.5/2 |

## RAW DATA FOR EXCEPTIONAL SUBJECTS IN CLASSROOM 5
(with group summary information)

| Subject | dig/min # | cntg % | add % | CTBS | B1 ●/X | B2 ●/X | B3 ●/X | B4 ●/X | B5 ●/X | B6 ●/X |
|---|---|---|---|---|---|---|---|---|---|---|
| 5.1 | 80 | 100 | 100 | 98 | 2/19 | 8/34 | 6/78 | 8/87 | 3/46 | 8/68 |
| Group Median | 51.5 | 100 | 100 | 87.5 | 2.5/16.5 | 5/30 | 5/39.5 | 6/42.5 | 3/43 | 5.5/46 |

| Subject | PRE % | C1 % ●/X | C2 % ●/X | C3 % ●/X | R4 % ●/X | R5 % ●/X | R6 % ●/X | MN7 ●/X | A8 % ●/X | A9 % ●/X | A10 % ●/X | POST % |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 5.1 | 5 | 100<br>2/7 | 100<br>4/2 | 100<br>9/5 | 80 | 80 | 70 |  | 60 | 90 | 90<br>11/0 | 60<br>75<br>90 |
| Group Median | 10 | 100<br>4/10 | 100<br>6/15.5 | 90<br>8.5/11 | 100<br>6.5/14.5 | 95<br>9/15.5 | 95<br>8.5/13.5 | 7/16.5 | 85<br>11/23 | 100<br>11.5/9 | 90<br>13/3 | 87.5 |

| Subject | 11 % ●/X | 12 % ●/X | 13 % ●/X | 14 % ●/X | 15 % ●/X | 16 % ●/X | 17 % ●/X | 18 % ●/X | 19 % ●/X | 20 % ●/X | 21 % |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 5.1 | 60<br>17/7 | 90<br>9/5 | 80 | 9/6 | 60<br>8/0 | 30<br>16/2 | 60<br>10/1 | 30<br>4/5 | 90<br>14/2 | 60<br>15/2 | 100<br>16/4 |
| Group Median | 90<br>13.5/4.5 | 90<br>11/9.5 | 80<br>15/2 | 80<br>9.5/2.5 | 80<br>14/0 | 80<br>16/1 | 80<br>8/3 | 90<br>14/1 | 90<br>17/0 | 90<br>13/4 | 100<br>18/2 |

### RAW DATA FOR EXCEPTIONAL SUBJECTS IN CLASSROOM 6
(with group summary information)

| Subject | dig/min<br>#| cntg<br>% | add<br>% | CTBS | B1<br>●/X | B2<br>●/X | B3<br>●/X | B4<br>●/X | B5<br>●/X | B6<br>●/X | B7<br>●/X | B8<br>●/X |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 6.1 | 43 | 100 | 100 | 88 | 7/5 | 3/33 | 3/51 | 3/51 | 1/53 | 1/53 | 0/54 | 1/53 |
| 6.2 | 80 | 100 | 100 | 46 | 3/43 | 4/40 | 2/48 | 1/53 | 0/53 | 1/53 | 1/53 | 1/53 |
| 6.3 | 50 | 100 | 95 | 90 | 2/23 | 1/58 | 0/54 | 1/53 | 0/54 | 1/53 | 0/54 | 1/53 |
| Group Median | 62 | 100 | 95 | 67 | 3/30 | 7.5/45.5 | 3/53 | 2/53 | 3/53 | 1/53 | 1/54 | 2/52 |

| Subject | PRE<br>% | C1<br>%<br>●/X | C2<br>%<br>●/X | C3<br>%<br>●/X | R4<br>%<br>●/X | R5<br>%<br>●/X | R6<br>%<br>●/X | MN7<br>●/X | A8<br>%<br>●/X | A9<br>%<br>●/X | A10<br>%<br>●/X | POST<br>% |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 6.1 | 20 | 100<br>2/52 | 100<br>14/43 | 100<br>14/41 | 100<br>17/37 | 100<br>15/39 | 90<br>21/37 | 11/28 | 70<br>17/18 | 100<br>15/3 | 100<br>12/6 | 70<br>100 |
| 6.2 | 0 | 10 100<br>5/49 | 100<br>1/36 | 100<br>5/49 | 100<br>6/42 | 100<br>6/33 | 80<br>10/54 | 3/44 | | 80<br>15/41 | 90<br>3/19 | 70<br>100 |
| 6.3 | 5 | 50 100<br>5/43 | 100<br>5/15 | 90<br>11/3 | 100<br>5/7 | 100<br>12/17 | 80<br>8/4 | 8/7 | 90<br>19/17 | 90<br>21/19 | 90<br>15/14 | 65<br>100 |
| Group Median | 5 | 70<br>3/52 | 100<br>6/39 | 100<br>8/23 | 100<br>7/14 | 100<br>9/8 | 90<br>7/14 | 5/0 | 90<br>11/1 | 90<br>8/1 | 100<br>8.5/0 | 85 |

| Subject | 11<br>%<br>●/X | 12<br>%<br>●/X | 13<br>%<br>●/X | 14<br>%<br>●/X | 15<br>%<br>●/X | 16<br>%<br>●/X | 17<br>%<br>●/X | 18<br>%<br>●/X | 19<br>%<br>●/X | 20<br>%<br>●/X | 21<br>%<br>●/X |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 6.1 | 80<br>10/2 | 80<br>13/0 | 90<br>23/1 | 100<br>5/4 | 100<br>17/1 | 90<br>16/2 | 90<br>16/1 | 90<br>20/0 | 80<br>24/0 | 80<br>24/0 | 26/0 |
| 6.2 | 60 | 16/1 | 90<br>12/2 | 50<br>7/0 | 80<br>8/0 | 80<br>12/0 | 90<br>10/1 | 80 | 70 | 70 | |
| 6.3 | 5/0 | 90<br>8/0 | 80<br>10/1 | 3/4 | 13/1 | | 9/2 | 100<br>12/3 | 17/3 | 70<br>18/2 | 70<br>18/2 |
| Group Median | 90<br>7/0 | 90<br>9/1 | 80<br>9/1 | 70<br>5/0 | 70<br>8/1 | 60<br>8/0 | 90<br>9/0 | 80<br>9/1 | 90<br>14/.5 | 85<br>17/0 | 90<br>14/0 |

# APPENDIX K
# RAW DATA FOR NORMALLY ACHIEVING SUBJECTS IN EACH CLASSROOM

PERCENTAGE AND RATE DATA FOR NORMALLY ACHIEVING SUBJECTS IN CLASSROOM 1

| PRE | C1 | C2 | C3 | R4 | R5 | R6 | MN7 | A8 | A9 | A10 | POST |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 80<br>2/18 | 70 80<br>1/7 | 40 80<br>3/18 | 90<br>4/13 | 90<br>6/18 | 70 90<br>7/3 | 4/9 | 70<br>7/0 | 100<br>12/4 | 90<br>12/1 | 70<br>100 |
| 10 | 100<br>2/5 | 90 | 100<br>3/0 | 100<br>5/2 | 100<br>2/0 | 90<br>6/2 | 5/0 | 100<br>8/0 | 100<br>7/1 | 90<br>11/0 | 80<br>95 |
| 35 | 100<br>8/6 | 100<br>11/0 | 100<br>14/0 | 80<br>17/0 | 100<br>12/0 | 100<br>16/0 | 10/1 | 100<br>13/4 | 100<br>19/1 | 100<br>22/0 | 95 |
| 10 | 80<br>3/2 | 100<br>7/2 | 80 | 90<br>5/4 | 90<br>10/1 | 100<br>13/1 | 6/1 | 90<br>13/1 | 90<br>17/1 | 100<br>17/1 | 75 |
| 0 | 80<br>4/7 | 70 100<br>1/8 | 60 90<br>10/5 | 80<br>10/10 | 70 90<br>12/18 | 80<br>10/15 | 7/5 | 80<br>15/3 | 80<br>21/10 | 90<br>20/0 | 55<br>100 |
| 10 | 100<br>5/8 | 80<br>3/1 | 100<br>3/0 | 100<br>0/2 | 90<br>4/0 | 90<br>4/5 | 5/5 | 80<br>14/7 | 80<br>16/11 | 90<br>16/0 | 50<br>90 |
| 15 | 80<br>3/0 | 90<br>4/2 | 90<br>8/0 | 90<br>5/2 | 90<br>2/0 | 100<br>8/0 | 5/0 | 80<br>8/0 | 80<br>5/2 | 100<br>16/0 | 90<br>100 |
| 0 | 100<br>5/0 | 80<br>4/0 | 80 | 100 | 100 | 100 | 3/2 | 80<br>10/0 | 100 |  |  |
| 35 | 50 100<br>3/7 | 100<br>6/3 | 90 | 100 | 90 | 15/8 | 9/19 | 100<br>5/0 | 90<br>14/12 | 90<br>12/2 | 90 |
| 40 | 100<br>8/12 | 90<br>9/0 | 100<br>10/0 | 90<br>6/2 | 90<br>12/0 | 100<br>13/3 | 6/1 | 90<br>10/2 | 100<br>17/0 | 100<br>22/0 | 95 |
| 10 | 100<br>6/15 | 90<br>4/20 | 90<br>6/19 | 100<br>7/0 | 90<br>8/5 | 90<br>9/2 | 7/0 | 90<br>3/1 | 100<br>5/1 | 90<br>12/2 | 80 |
| 15 | 90<br>3/2 | 90 | 90 | 100<br>8/0 | 90<br>7/1 | 90<br>9/1 | 7/0 | 80<br>10/3 | 100<br>11/11 | 90<br>14/0 | 85<br>100 |
| 15 | 100<br>5/6 | 80<br>12/1 | 60 90<br>8/3 | 100<br>10/14 | 90<br>12/15 | 100<br>13/8 | 12/11 | 100<br>18/6 | 100<br>19/10 | 90<br>18/4 | 70<br>100 |
| 15 | 90<br>2/7 | 100<br>4/2 | 100<br>8/0 | 100<br>3/4 | 90<br>2/3 | 100<br>8/0 | 7/0 | 90<br>11/1 | 100<br>8/2 | 100<br>15/1 | 80<br>90 |
| 20 | 90 | 70 100<br>8/7 | 100<br>12/2 | 100<br>5/2 | 100<br>12/2 | 100<br>13/3 | 15/4 | 90<br>15/2 | 90<br>21/4 | 100<br>16/0 | 80<br>90 |
| 10 | 80<br>2/11 | 60 90<br>2/18 | 60 80<br>2/11 | 90 | 1/2 |  | 4/1 | 90<br>5/9 | 90<br>8/0 | 90<br>4/5 | 85<br>90 |

PERCENTAGE AND RATE DATA FOR NORMALLY ACHIEVING SUBJECTS IN CLASSROOM 1--CONTINUED

| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
|---|---|---|---|---|---|---|---|---|---|---|
| 100<br>10/2 | 100<br>6/1 | 60<br>8/1 | 80<br>12/3 | 100<br>7/3 | 50<br>14/1 | 90<br>14/1 | 50<br>8/3 | 90<br>14/0 | 70<br>12/0 | 100 |
| 90<br>12/0 | 90<br>12/2 | 70<br>15/1 | 60<br>14/4 | 70<br>9/2 | 90<br>16/2 | 70<br>15/1 | 70<br>16/1 | 80<br>20/2 | 80<br>11/1 | 90 |
| 100<br>12/0 | 100<br>17/2 | 90<br>20/2 | 90<br>31/0 | 100<br>7/2 | 100<br>31/0 | 100 | 100<br>19/1 | 80<br>20/0 | 80<br>21/1 | 100 |
| 80<br>11/1 | 80<br>12/3 | 100 |  | 7/0 | 70<br>16/0 | 80<br>18/2 | 40<br>9/4 | 90<br>18/4 | 80<br>17/2 | 100 |
| 100<br>11/1 | 100<br>16/2 | 60<br>19/3 | 70 | 50<br>13/4 | 50<br>16/8 | 60<br>16/4 | 70<br>14/4 | 50<br>15/9 | 70<br>18/6 | 100 |
| 60<br>11/1 | 100<br>10/7 | 70<br>15/1 | 40<br>14/0 | 70<br>6/5 | 10/0 | 90<br>15/1 | 60<br>7/3 | 60<br>11/1 | 80<br>11/1 |  |
| 90<br>11/1 | 100<br>17/1 | 100<br>20/0 | 70<br>18/0 | 70<br>8/1 | 60<br>17/0 | 90<br>21/1 | 13/2 | 100<br>22/0 | 70<br>12/0 | 100 |
| 12/0 | 100<br>7/0 | 80<br>15/1 | 14/0 | 80<br>15/0 | 100<br>18/0 |  |  | 90<br>18/0 | 60<br>15/2 | 100 |
| 100 | 2/0 | 70<br>7/2 | 80 | 5/0 | 100<br>8/0 | 100<br>14/3 |  | 80<br>22/0 | 70<br>19/0 |  |
| 100<br>12/0 | 100<br>18/0 | 100<br>22/0 | 100<br>26/0 | 90<br>11/0 | 100<br>31/0 | 100<br>22/0 | 80<br>17/0 | 90<br>28/0 | 90<br>24/0 | 100 |
| 90<br>12/2 | 70<br>10/8 | 70<br>19/1 | 50 | 50<br>9/6 | 70<br>12/2 | 100<br>22/0 | 90<br>10/0 | 100<br>20/0 | 60<br>13/4 | 80 |
| 100 | 100<br>10/0 | 90<br>13/1 | 100 | 100<br>11/0 | 70<br>10/3 | 90<br>16/1 | 60<br>15/0 | 80<br>15/1 | 80<br>12/0 | 100 |
| 100<br>17/2 | 100<br>17/2 | 100<br>22/0 | 40<br>30/1 | 60<br>15/2 | 80<br>31/0 | 70<br>22/0 | 90<br>12/2 | 90<br>22/0 | 80<br>18/6 | 100 |
| 100<br>12/0 | 90<br>17/0 | 80<br>9/0 | 80<br>18/0 | 90<br>7/0 | 90<br>18/0 | 70<br>19/0 | 70<br>7/1 | 90<br>20/0 | 80<br>23/0 | 90 |
| 90<br>15/4 | 90<br>16/5 | 90<br>20/2 | 70<br>31/2 | 90 |  | 80<br>27/4 | 60<br>24/4 | 90<br>20/2 | 80<br>17/7 | 100 |
| 7/1 | 70<br>6/2 | 70<br>9/2 | 70<br>15/1 | 90<br>6/1 | 40<br>11/3 |  |  | 60<br>8/1 | 70<br>8/0 | 50 |

_Note_: Percentage scores are recorded over rate (correct digits/incorrect digits) data.

## PERCENTAGE AND RATE DATA FOR NORMALLY ACHIEVING SUBJECTS IN CLASSROOM 2

| PRE | C1 | C2 | C3 | R4 | R5 | R6 | MN7 | A8 | A9 | A10 | POST |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 45 | 100<br>10/9 | 100<br>10/5 | 80<br>8/31 | 100<br>10/21 | 90<br>5/52 | 100<br>11/5 | 6/1 | 90<br>15/4 | 100<br>11/11 | 100<br>16/0 | 80<br>90 |
| 35 | 100<br>11/9 | 100<br>10/40 | 90<br>16/2 | 100<br>13/28 | 100<br>15/17 | 90<br>16/0 | 5/0 | 100<br>2/0 | 90<br>2/0 | 100<br>14/2 | 75<br>95 |
| 15 | 100<br>14/47 | 100<br>13/0 | 100<br>10/0 | 100<br>16/41 | 100<br>8/9 | 100<br>8/0 | 5/0 | 90<br>11/1 | 100<br>6/2 | 100<br>14/0 | 80<br>90 |
| 15 | 100<br>6/8 | 100<br>9/0 | 80<br>7/3 | 100<br>4/6 | 100<br>12/0 | 90<br>12/2 | 7/6 | 100<br>15/6 | 100<br>5/5 | 100<br>16/4 | 85<br>100 |
| 5 | 100 | 100<br>3/1 | 80<br>0/9 | 100<br>1/1 | 90<br>0/2 | 100<br>5/3 | 4/2 | 100<br>4/1 | 100<br>5/0 | 100<br>8/0 | 80<br>95 |
| 35 | 100<br>6/8 | 100<br>7/7 | 90<br>9/13 | 100<br>9/15 | 100<br>10/8 | 90<br>11/3 | 5/6 | 80<br>12/7 | 100<br>10/2 | 100<br>12/1 | 75<br>90 |
| 10 | 100<br>6/17 | 100<br>10/2 | 90<br>8/0 | 100<br>6/8 | 90<br>9/11 | 90<br>7/1 | 5/0 | 80<br>10/2 | 100<br>7/1 | 100<br>17/1 | 70<br>90 |
| 10 | 100<br>3/4 | 90 | 100<br>6/2 | 100<br>3/7 | 80<br>7/2 | 80<br>7/1 | 3/1 | 100<br>7/5 | 100<br>7/3 | 100<br>9/0 | 60<br>95 |
| 10 | 100<br>4/4 | 100<br>9/2 | 100<br>6/2 | 100<br>8/9 | 100<br>8/4 | 80<br>14/0 | 5/5 | 100<br>11/10 | 100<br>6/8 | 100 | 85<br>100 |
| 30 | 80<br>4/4 | 90<br>6/3 | 90<br>8/1 | 100<br>5/5 | 100<br>8/6 | 100<br>8/0 | 2/0 | 80<br>0/4 | 80<br>5/3 | 100<br>8/1 | 85<br>95 |
| 5 | 90<br>4/16 | 100<br>1/3 | 100<br>5/14 | 100<br>5/14 | 100<br>5/15 | 100<br>4/14 | 4/14 | 100<br>10/13 | 100<br>9/17 | 100<br>10/9 | 75<br>90 |
| 5 | 100<br>3/2 | 80<br>7/5 | 100<br>8/22 | 80<br>9/16 | 100<br>12/35 | 80<br>3/2 | 3/0 | 70 100<br>6/4 | 50 100<br>6/12 | 100<br>6/4 | 65<br>90 |
| 20 | 100<br>4/7 | 100<br>11/2 | 80<br>9/10 | 100<br>10/21 | 60 80<br>9/10 | 80<br>8/4 | 6/4 | 90<br>10/6 | 90<br>9/8 | 80<br>9/4 | 60<br>95 |
| 15 | 90<br>2/1 | 100<br>5/3 | 100<br>5/6 | 100<br>5/14 | 100<br>8/15 | 100<br>7/0 | 3/0 | 90<br>2/0 | 90<br>2/0 | 100<br>8/1 | 90 |
| 5 | 100<br>4/1 | 80<br>7/14 | 90<br>3/2 | 100<br>6/12 | 70 100<br>5/14 | 100<br>7/18 | 9/12 | 100<br>8/8 | 100<br>11/7 | 100<br>7/4 | 65<br>90 |
| 40 | 100<br>10/5 | 100<br>17/1 | 90<br>15/1 | 100<br>12/6 | 100<br>17/4 | 100<br>18/0 | 8/1 | 90<br>12/0 | 100<br>17/1 | 100<br>17/3 | 75<br>90 |
| 10 | 100<br>3/33 | 100<br>7/2 | 90<br>3/36 | 100<br>4/30 | 80<br>6/3 | 80<br>6/10 | 2/3 | 100<br>7/7 | 100<br>10/21 | 100<br>6/7 | 65<br>90 |

PERCENTAGE AND RATE DATA FOR NORMALLY ACHIEVING SUBJECTS IN CLASSROOM 2--CONTINUED

| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
|---|---|---|---|---|---|---|---|---|---|---|
| 100 | 90<br>10/0 | 100<br>20/1 | 90<br>7/2 | 70<br>18/0 | 90<br>24/0 | 90<br>11/0 | 19/1 | 100<br>22/0 | 90<br>22/1 | 100<br>25/2 |
| 90<br>12/0 | 90<br>15/0 | 90<br>22/0 | 80<br>7/0 | 60<br>19/1 | 90<br>24/0 | 90<br>9/2 | 80<br>12/3 | 100<br>22/0 | 100<br>19/2 | 90<br>22/2 |
| 100<br>12/0 | 100<br>18/0 | 100<br>15/1 |  | 90<br>8/0 | 100<br>16/2 | 90<br>10/1 | 50<br>10/0 | 80<br>16/0 | 100<br>12/0 | 80<br>16/0 |
| 100<br>16/1 | 100<br>14/1 | 100<br>22/0 | 80<br>7/2 | 70<br>22/0 | 90<br>31/0 | 90<br>5/2 | 90<br>6/2 | 90<br>22/0 | 90<br>16/1 | 100<br>22/0 |
| 90<br>11/1 | 100<br>8/0 | 100<br>11/0 |  | 80<br>8/0 | 90<br>8/0 | 90<br>5/0 | 90<br>10/0 | 100<br>16/0 | 100<br>12/0 | 100<br>9/0 |
| 100<br>18/3 | 100<br>5/7 | 100<br>17/1 | 90<br>7/0 | 90<br>18/0 | 90<br>20/0 | 90<br>7/0 | 50<br>18/0 | 80<br>19/1 | 100<br>12/0 | 100<br>12/2 |
| 90<br>12/0 | 100<br>7/1 | 100<br>20/0 | 70<br>8/1 | 50<br>18/0 | 80<br>28/1 | 80<br>6/1 | 60<br>14/1 | 21/1 | 70<br>12/0 | 100<br>21/1 |
| 80<br>12/0 | 100<br>12/3 | 50<br>5/1 | 80<br>6/3 | 60<br>5/2 | 40<br>14/0 | 80<br>5/2 | 90<br>5/5 | 70<br>10/3 | 90<br>10/2 | 80<br>9/0 |
| 100<br>12/0 | 100<br>9/1 | 100<br>16/0 | 100<br>5/4 | 100<br>20/0 | 100<br>18/0 | 100<br>5/0 |  |  |  | 100<br>14/0 |
| 90 | 0/2 | 70<br>8/0 | 50<br>2/0 | 100<br>7/0 | 80<br>8/0 | 90<br>5/0 | 70<br>2/0 | 90<br>10/1 | 100<br>9/1 | 100<br>9/0 |
| 90<br>9/12 | 90<br>9/8 | 11/4 | 50<br>3/5 |  |  | 80<br>6/5 | 80<br>3/6 | 80<br>14/2 | 90<br>13/6 | 90<br>14/2 |
| 90<br>7/3 | 100<br>3/1 | 60<br>3/1 | 70<br>3/1 | 60<br>3/0 | 80<br>7/1 | 80<br>9/11 | 60<br>5/3 | 70<br>9/0 | 100<br>9/3 | 80<br>7/4 |
| 70<br>14/9 | 100<br>11/6 | 50<br>11/0 | 50<br>6/5 | 50<br>14/0 | 100<br>14/0 | 70<br>7/2 | 60<br>9/3 | 70<br>14/2 | 70<br>16/1 | 90<br>20/2 |
| 100<br>2/0 | 100<br>10/0 | 90<br>13/0 | 90<br>7/0 | 90<br>14/0 | 100<br>18/0 | 90<br>5/2 | 70<br>10/0 | 100<br>19/1 | 100<br>17/0 | 90<br>20/0 |
| 90<br>9/3 | 100<br>8/7 | 90<br>9/0 | 70<br>5/6 | 80<br>5/2 | 80<br>8/0 | 70<br>3/2 | 90<br>10/0 | 90<br>10/1 | 90<br>11/1 | 90<br>9/0 |
| 100<br>16/3 | 100<br>18/0 | 100<br>21/2 | 100<br>7/0 | 90<br>24/0 |  | 90<br>7/0 | 100<br>18/0 | 100<br>22/0 | 100<br>17/0 | 90<br>22/0 |
| 100<br>7/10 | 80<br>12/8 | 40<br>10/7 | 80<br>5/12 | 70<br>7/7 | 70<br>12/6 | 60<br>7/7 | 50<br>7/3 | 70<br>13/3 | 80<br>13/6 | 90<br>15/1 |

Note: Percentage scores are recorded over rate (correct digits/incorrect digits) data.

PERCENTAGE AND RATE DATA FOR NORMALLY ACHIEVING SUBJECTS IN CLASSROOM 3

| PRE | C1 | C2 | C3 | R4 | R5 | R6 | MN7 | A8 | A9 | A10 | POST |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 10 | 80<br>4/8 | 90<br>9/1 | 100 | 100<br>3/1 | 90<br>12/3 | 100<br>7/3 | 5/2 | 90<br>9/5 | 100<br>10/3 | 90<br>9/0 | 85<br>95 |
| 50 | 80<br>3/6 | 80<br>12/1 | 60 100<br>9/1 | 100<br>7/1 | 80 | 100<br>6/3 | 3/0 | | | | 80 90<br>100 |
| 55 | 90<br>5/6 | 100<br>10/3 | 100<br>10/3 | 100<br>9/5 | 90<br>13/2 | 90<br>12/1 | 7/4 | 90<br>13/2 | 90<br>10/2 | 100<br>16/3 | 85<br>95 |
| 10 | 100<br>1/5 | 90<br>9/4 | 90<br>2/9 | 100<br>7/11 | 80<br>10/9 | 100<br>7/2 | 5/0 | 70 100<br>13/5 | 100<br>14/10 | 100<br>14/8 | 80 90<br>100 |
| 15 | 10 100<br>3/9 | 100<br>4/3 | 80<br>6/7 | 100<br>5/10 | 90<br>9/14 | 90<br>12/1 | 7/0 | 100<br>13/7 | 100<br>17/10 | 80<br>15/6 | 100 |
| 55 | 90<br>3/2 | 100<br>9/0 | 90<br>5/0 | 100<br>4/5 | 90<br>10/1 | 100<br>8/0 | 5/0 | 100<br>12/0 | 100<br>7/1 | | 100 |
| 20 | 40 100<br>7/44 | 100<br>6/23 | 80<br>4/28 | 90<br>3/5 | 70 100<br>8/4 | 100<br>8/0 | 4/1 | 70 100<br>10/2 | 100<br>5/4 | 90<br>12/1 | 65<br>90 |
| 0 | 90 | 100 | 90<br>0/3 | 100 | 100<br>2/0 | 100<br>5/0 | 2/1 | 60 100<br>7/14 | 100<br>12/20 | 90<br>8/3 | 80<br>100 |
| 15 | 80 | 100 | 50 100<br>5/2 | 100<br>0/4 | 80<br>0/5 | 100<br>3/4 | 3/1 | 90<br>8/3 | 80<br>11/2 | 90<br>14/1 | 80 80<br>90 |
| 10 | 100 | 90 | 80 | 100<br>2/0 | 100<br>12/0 | 90<br>7/2 | 5/0 | 100<br>2/0 | 100<br>8/1 | 90<br>14/2 | 70<br>90 |
| 60 | 90<br>5/0 | 80<br>11/0 | 100 | 100<br>2/4 | 100<br>4/1 | 100<br>7/1 | 7/0 | 80<br>8/0 | 100<br>2/2 | 100<br>14/0 | 75<br>90 |
| 65 | 100<br>10/14 | 90<br>8/12 | 90<br>11/17 | | | | | 100<br>9/12 | 100<br>10/2 | 90<br>12/2 | 85<br>100 |
| 70 | 90<br>9/9 | 100 | 100<br>14/5 | 100<br>2/0 | 80<br>15/6 | 100<br>14/5 | | 100<br>17/10 | 100<br>13/8 | 100<br>9/0 | 90 |
| 15 | 10 100<br>3/5 | 90<br>3/2 | 50 100<br>3/10 | 80<br>4/18 | 60 100<br>6/23 | 80<br>6/19 | 3/0 | 80<br>8/19 | 90<br>5/1 | 100<br>5/17 | 70<br>90 |
| 5 | 80<br>2/5 | 100<br>12/1 | 70 100<br>7/2 | 90<br>2/5 | 90<br>12/5 | 40 100<br>9/13 | 4/1 | 80<br>1/4 | 90<br>9/8 | 80<br>11/5 | 45 70<br>100 |
| 20 | 100<br>7/14 | 100<br>11/0 | 90 | 100<br>14/10 | 80<br>15/8 | 100<br>8/2 | 8/3 | 100<br>19/5 | 100<br>16/14 | 100<br>16/9 | 95 |
| 5 | 90<br>3/22 | 100<br>6/6 | 70<br>8/24 | 100 | 60 100 | 60 100<br>6/4 | 3/0 | 90<br>7/13 | 100<br>12/22 | 70 90 | 65 90<br>100 |
| 5 | 100<br>5/0 | 80<br>8/1 | 90<br>4/2 | 100<br>3/1 | 70 100<br>5/2 | 80<br>8/2 | 3/0 | 80<br>12/2 | 90<br>9/3 | 80<br>9/0 | 85<br>90 |
| 30 | 100<br>5/2 | 90<br>8/3 | 90<br>11/3 | 100<br>4/3 | 90<br>14/3 | 100<br>8/2 | 5/0 | 90<br>10/2 | 100<br>7/1 | 90<br>10/1 | 60<br>90 |

PERCENTAGE AND RATE DATA FOR NORMALLY ACHIEVING SUBJECTS IN CLASSROOM 3--CONTINUED

| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
|---|---|---|---|---|---|---|---|---|---|---|
| 100<br>12/0 | 80<br>6/1 | 80<br>9/0 | 90<br>5/0 | 90<br>14/0 | 90<br>18/0 | 80<br>15/1 | 80<br>12/0 | 80<br>14/1 | 90<br>12/0 | 90<br>20/0 |
| 100<br>12/0 | 60<br>15/3 | 60<br>20/2 | 60<br>9/8 | 60<br>18/4 | 80<br>23/1 | 70<br>26/10 | 50<br>20/8 | 90<br>28/10 | 60<br>21/7 | 100<br>34/9 |
| 100<br>12/0 | 100<br>8/2 | 70<br>22/0 | 60<br>9/4 | 50<br>14/0 | 70<br>18/0 | 90<br>11/0 | 70<br>18/0 | 90<br>22/0 | 100<br>10/2 | 90<br>14/0 |
| 100<br>12/0 | 100<br>13/3 | 50<br>18/0 | 70<br>10/3 | 80<br>16/0 | 70<br>17/1 | 80<br>9/0 | 60<br>17/0 | 70<br>17/1 | 100<br>12/0 | 90<br>18/2 |
| 100<br>15/1 | 100<br>16/2 | 100<br>22/0 | 70<br>11/1 | 90<br>18/0 | 50<br>26/0 | 90<br>17/4 | 70<br>14/3 | 80<br>24/1 | 90<br>16/1 | 90<br>20/1 |
| 100<br>12/0 | 100<br>13/2 | 100<br>21/1 | 100<br>7/0 | 70<br>12/2 | 90<br>15/3 | 90<br>7/0 | 100<br>14/2 | 70<br>16/6 | 100<br>10/2 | 100<br>15/1 |
| 100<br>11/0 | 100<br>13/2 | 80<br>18/0 | 90<br>11/2 | 60<br>16/1 | 60<br>18/0 | 80<br>16/1 | 40<br>11/1 | 80<br>21/1 | 90<br>9/3 | 100<br>13/3 |
| 80<br>11/1 | 100<br>4/0 | 70<br>14/1 | 70<br>9/4 | 70<br>12/2 | 60<br>14/4 | 60<br>15/3 | 60<br>17/3 | 90<br>17/6 | 70<br>11/3 | 90<br>18/2 |
| 90<br>11/1 | 90<br>3/1 | 60<br>14/0 | 100<br>7/0 | 50<br>8/0 | 50<br>18/0 | 70<br>11/0 | 60<br>18/2 | 80<br>16/2 | 90<br>12/0 | 90<br>8/1 |
| 90<br>12/0 | 100<br>13/2 | 80<br>14/0 | 60<br>7/0 | 40<br>14/0 | 50<br>17/1 | 50<br>7/0 | 60<br>16/6 | 60<br>18/6 | 70 | 60<br>14/3 |
| 100<br>8/0 | 90<br>2/0 | 80<br>11/0 | 60<br>5/0 | 90<br>14/0 | 90<br>10/0 | 90<br>4/2 | 80<br>7/1 | 80<br>7/1 | 100<br>7/0 | 100<br>9/0 |
| 80<br>8/0 | 100<br>8/0 | 100<br>9/0 | 7/0 | 70<br>16/0 | 60<br>16/0 | 90<br>11/3 | 80<br>14/7 | 90<br>20/1 | 100<br>17/3 | 100<br>9/0 |
| 90<br>12/0 | 100<br>8/2 | 100<br>22/0 | 80<br>7/0 | 100<br>18/0 | 100<br>24/0 | 100<br>7/0 | 100<br>18/1 | 90<br>23/1 | 100<br>21/0 | 90<br>20/0 |
| 90<br>4/0 | 90<br>8/4 | 60<br>13/2 | 60<br>4/7 | 40<br>7/1 | 60<br>11/1 | 80<br>8/7 | 100<br>16/10 | 80<br>10/4 | 70<br>15/2 | 80<br>9/1 |
| 90<br>11/1 | 80<br>10/8 | 70<br>14/0 | 70<br>4/10 | 70<br>14/0 | 50<br>17/3 | 70<br>10/9 | 70<br>13/4 | 60<br>16/5 | 90<br>12/2 | 70<br>12/1 |
| 90<br>12/0 | 100<br>8/0 | 100<br>20/0 | 70 | 90<br>24/0 | 90<br>19/1 | 90<br>10/3 | 80<br>16/2 | 90<br>24/0 | 100<br>29/2 | 90<br>24/0 |
| 90<br>5/0 | 80<br>12/3 | 70<br>8/1 | 60<br>4/4 | 60<br>5/0 | 40<br>12/1 | 80<br>6/2 | 40<br>0/2 | 70<br>11/0 | 100<br>11/4 | 80<br>9/0 |
| 90<br>12/0 | 100<br>6/2 | 100<br>16/0 | 80 | 70<br>8/0 | 90<br>18/0 | 60<br>8/7 | 60<br>13/0 | 80<br>20/0 |  |  |
| 90<br>15/6 | 100<br>14/4 | 70<br>13/1 | 40<br>9/6 | 30<br>5/0 | 100<br>10/0 | 60<br>9/8 | 30<br>16/2 | 60<br>20/4 | 90<br>17/6 | 100<br>13/4 |

Note: Percentage scores are recorded over rate (correct digits/incorrect digits) data.

PERCENTAGE AND RATE DATA FOR NORMALLY ACHIEVING SUBJECTS IN CLASSROOM 4

| PRE | C1 | C2 | C3 | R4 | R5 | R6 | MN7 | A8 | A9 | A10 | POST |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 20 | 100<br>5/9 | 90<br>10/4 | 90<br>2/7 | 100<br>8/19 | 100<br>13/14 | 90<br>10/10 | 11/6 | 100 | 100<br>16/10 | 100<br>16/7 | 100 |
| 30 | 80<br>5/8 | 100<br>8/4 | 80<br>5/10 | 100<br>9/7 | 6/7 | 100<br>12/6 | 10/8 | 100 | 100<br>9/10 | 100<br>12/12 | 100 |
| 0 | 50 100<br>4/0 | 80<br>5/5 | 100<br>5/12 | 90<br>13/15 | 70 100<br>6/6 | 100<br>8/0 | 6/6 | 100 | 50 70<br>90<br>4/5 | 100<br>10/5 | 90 |
| 25 | 80<br>1/13 | 90 | 100<br>9/4 | 100<br>8/15 | 90<br>9/11 | 70 100<br>9/12 | 2/9 | 80 | 80<br>6/13 | 70 | 80<br>90 |
| 20 | 100<br>5/4 | 90<br>8/1 | 100<br>6/26 | 100<br>12/26 | 100<br>12/32 | 100<br>10/40 | 9/3 | 100 | 100<br>17/28 | 100<br>19/27 | 85<br>100 |
| 25 | 90<br>6/16 | 90<br>4/14 | 80<br>9/11 | 100<br>16/19 | 80<br>12/13 | 90<br>12/11 | 5/13 | 100 | 100<br>9/8 | 80<br>11/13 | 80<br>100 |
| 15 | 100<br>4/2 | 100<br>9/2 | 100<br>7/1 | 100<br>11/3 | 100 | 100<br>7/3 | 5/1 | 90 | 100<br>8/6 | 100<br>14/3 | 95 |
| 15 | 100<br>3/11 | 70 90 | 80<br>8/6 | 100<br>2/14 | 90<br>8/19 | 100<br>7/8 | 2/26 | 90 | 100<br>14/21 | 90<br>9/22 | 65<br>85 |
| 20 | 100<br>2/51 | 80 | 90<br>5/41 | 100<br>10/44 | 80<br>7/50 | 90<br>8/18 | 8/19 | 90 | 100<br>13/21 | 90<br>11/17 | 90 |
| 15 | 100<br>5/11 | 100<br>6/24 | 90<br>6/23 | 100<br>11/19 | 100<br>12/17 | 90<br>15/20 | 3/10 | 80 | 90<br>12/21 | 90<br>12/17 | 100 |
| 20 | 100<br>7/2 | 100<br>8/3 | 90<br>14/17 | 100 | 100<br>12/11 | 100<br>13/11 | 20/19 | 90 | 100<br>19/19 | 100<br>26/18 | 100 |
| 15 | 90<br>7/4 | 90<br>8/4 | 90<br>6/8 | 100<br>6/10 | 100<br>9/8 | 90<br>10/6 |  | 90 | 100<br>6/9 | 100<br>14/4 | 95 |
| 50 | 100 | 90<br>4/25 | 90<br>12/30 | 100<br>20/24 | 90<br>9/30 | 80<br>17/18 | 10/32 | 90 | 100<br>19/22 | 100 | 90 |
| 30 | 100<br>9/15 | 100<br>17/11 | 100<br>12/6 | 100<br>14/11 | 100<br>14/17 | 80<br>17/13 | 7/20 | 90 | 80<br>17/14 | 100<br>20/11 | 95 |
| 15 | 70 100<br>5/8 | 80<br>6/13 | 70 90<br>6/14 | 100<br>5/20 | 80<br>6/9 | 80<br>6/14 | 6/18 | 70 90 | 80<br>4/14 | 100<br>10/5 | 90 |
| 20 | 80<br>6/30 | 70 100<br>9/27 | 80 | 100<br>19/35 | 90<br>16/42 | 60<br>10/19 | 8/25 | 70 80 | 100<br>14/13 | 100<br>17/11 | 90 |

PERCENTAGE AND RATE DATA FOR NORMALLY ACHIEVING SUBJECTS IN CLASSROOM 4--CONTINUED

| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
|---|---|---|---|---|---|---|---|---|---|---|
| 100<br>12/0 | 80<br>16/7 | 50<br>18/0 | 90<br>10/1 | 100<br>16/0 | 100<br>16/3 | 100<br>12/9 | 90<br>10/4 | 100<br>22/0 | 100<br>19/2 | 100<br>21/1 |
| 100<br>4/0 | 100<br>8/0 | 70<br>15/2 | 90 |  | 100<br>22/0 | 90<br>9/0 | 80<br>17/0 |  |  |  |
| 70<br>6/10 | 100<br>9/13 | 70<br>13/5 | 60<br>3/1 | 100<br>7/7 | 100<br>7/13 | 90<br>7/10 | 60<br>12/11 | 90<br>15/13 | 90<br>9/18 | 60<br>12/12 |
| 100<br>11/1 | 100<br>7/4 | 70<br>9/2 | 80<br>6/1 | 100<br>18/0 | 100<br>18/0 | 100<br>17/0 | 90<br>18/1 | 100<br>16/4 | 90<br>22/2 | 60<br>22/0 |
| 100<br>12/0 | 100<br>10/0 | 90<br>8/2 | 80<br>7/0 | 100<br>18/0 | 100<br>16/4 | 100<br>11/2 | 100<br>12/3 | 100<br>17/1 | 100<br>12/0 | 100<br>19/3 |
| 90<br>11/3 | 90<br>10/12 | 60<br>9/2 | 90<br>10/5 | 100<br>18/0 | 80<br>24/0 | 100<br>8/5 | 80<br>14/4 | 100 | 90 | 80 |
| 100<br>12/0 | 100<br>10/5 | 100 | 100<br>15/2 | 100<br>20/0 | 100<br>12/4 | 100<br>11/1 | 100<br>15/0 | 100<br>22/0 | 100<br>16/1 | 90<br>22/0 |
| 80<br>3/1 | 90<br>5/2 | 60<br>9/0 | 80<br>5/0 | 90<br>12/2 | 90<br>9/2 | 100<br>5/0 | 100<br>2/0 | 90<br>7/2 | 80<br>7/1 | 70<br>8/1 |
| 100<br>2/0 | 90<br>11/12 | 60<br>10/5 | 70<br>14/8 | 100<br>16/0 | 80<br>10/0 | 100<br>9/5 | 100<br>9/11 | 90<br>9/5 | 90<br>11/8 | 70<br>15/2 |
| 100 | 100<br>10/18 | 13/5 | 40<br>13/4 | 100<br>14/4 | 100<br>17/15 | 100<br>14/17 | 80<br>15/14 | 100<br>17/5 | 100<br>25/10 | 70<br>21/12 |
| 100<br>16/8 | 90<br>15/9 | 70<br>38/2 | 90<br>38/2 | 100<br>28/5 | 90<br>33/2 | 100<br>11/0 | 90<br>20/3 | 100<br>22/0 | 100<br>23/1 | 100<br>23/1 |
| 100<br>10/2 | 90<br>8/2 | 70<br>17/1 | 60<br>9/11 | 100<br>20/0 | 90<br>18/0 | 100<br>7/0 | 90<br>7/1 | 100<br>17/5 | 100<br>12/0 | 21/1 |
| 100<br>13/2 |  | 19/3 | 50<br>17/5 | 90<br>22/4 | 100<br>26/5 | 100<br>18/11 | 80<br>18/10 | 90<br>37/12 | 90<br>20/9 | 80<br>24/8 |
| 80<br>10/4 | 100<br>15/2 | 70<br>18/0 | 70<br>6/1 | 100<br>31/0 | 100<br>24/0 | 100<br>8/7 | 90<br>18/0 | 100<br>22/0 | 100<br>22/2 | 100<br>20/2 |
| 100<br>6/11 | 100<br>5/20 | 60<br>12/3 | 60<br>4/9 | 100<br>16/0 | 80<br>11/5 | 90<br>6/11 | 70<br>8/9 | 100<br>14/8 | 80<br>13/4 | 90<br>12/4 |
| 100<br>12/0 | 100<br>7/1 | 70<br>14/0 | 70 |  | 80<br>17/1 | 80<br>9/8 | 16/2 | 90<br>19/3 | 100<br>19/5 | 90<br>19/3 |

Note: Percentage scores are recorded over rate (correct digits/incorrect digits) data.

## PERCENTAGE AND RATE DATA FOR NORMALLY ACHIEVING SUBJECTS IN CLASSROOM 5

| PRE | C1 | C2 | C3 | R4 | R5 | R6 | MN7 | A8 | A9 | A10 | POST |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 20 | 100<br>7/7 | 90<br>14/1 | 90<br>14/8 | 100<br>6/7 | 100<br>10/4 | 80<br>12/2 | 9/6 | 90<br>11/1 | 100<br>12/2 | 100<br>20/0 | 90 |
| 5 | 90<br>1/2 | 90 | 80<br>12/36 | 100<br>6/24 | 100<br>7/38 | 90<br>5/0 | 4/2 | 80<br>1/43 | 90<br>4/1 | 100<br>6/3 | 85<br>95 |
| 5 | 90<br>3/32 | 90<br>2/20 | 70 80<br>5/4 | 100<br>3/18 | 100<br>15/38 | 100<br>6/19 | 2/55 | 90<br>15/39 | 100<br>11/18 | 9/3 | 80<br>95 |
| 10 | 100<br>6/1 | 90<br>4/1 | 100<br>7/1 | 100<br>5/4 | 90<br>7/1 | 80<br>7/10 | 7/14 | 80<br>12/13 | 100<br>14/11 | 100<br>17/2 | 75 70<br>95 |
| 40 | 100<br>5/0 | 100<br>6/13 | 90 | 100<br>1/12 | 70 100<br>3/53 | 80<br>7/62 | 7/58 | 80<br>4/61 |  | 70 100 | 95 |
| 10 | 90<br>3/12 | 90 | 70 100<br>9/21 | 100<br>5/22 | 90<br>6/20 | 90<br>3/22 | 5/17 | 90<br>8/17 | 100<br>9/5 | 100<br>12/2 | 60<br>90 |
| 40 | 100<br>3/4 | 100<br>18/22 | 100<br>22/21 | 100<br>16/22 | 90<br>22/26 | 100<br>21/19 | 13/20 | 90<br>21/17 |  | 100<br>13/3 | 95 |
| 0 | 90 | 90<br>2/18 | 100 | 100<br>9/17 | 90<br>8/15 | 100<br>10/20 | 7/17 | 90<br>9/25 | 100<br>14/14 | 90<br>14/10 | 95 |
| 0 | 100<br>4/6 | 100<br>6/46 | 100<br>6/12 | 100<br>11/19 | 100<br>12/29 | 100<br>9/52 | 7/47 | 90<br>10/38 | 100<br>6/6 | 90<br>9/0 | 90 |
| 15 | 100<br>4/10 | 100<br>12/11 | 90<br>11/10 | 100<br>9/10 | 70 100<br>11/16 | 90<br>6/7 | 5/6 | 80<br>14/17 | 100<br>15/10 | 70 100<br>6/3 | 85<br>95 |
| 40 | 90<br>7/10 | 100<br>21/32 | 100<br>15/17 | 90<br>15/9 | 100<br>14/13 | 100<br>13/5 | 17/40 | 100<br>24/30 | 100<br>13/11 | 100<br>21/1 | 90 |
| 40 | 100<br>6/1 | 90<br>10/4 | 90<br>5/9 | 100<br>2/5 | 100<br>15/4 | 100<br>10/3 | 9/6 | 80<br>16/5 | 100<br>13/1 | 100<br>16/2 | 95 |
| 10 | 90<br>2/13 | 100<br>3/11 | 90<br>4/8 | 100<br>5/7 | 100<br>6/13 | 70 90<br>5/13 | 3/4 | 80<br>7/12 | 100<br>8/5 | 80<br>14/0 | 75 85<br>95 |
| 10 | 100<br>4/11 | 100<br>5/7 | 70 100<br>5/6 | 100<br>8/9 | 100<br>7/10 | 100<br>10/14 | 9/16 | 100<br>12/23 | 100<br>18/30 | 90<br>10/14 | 90 |
| 10 | 80<br>0/33 | 90<br>7/47 | 60 90<br>3/46 | 100<br>3/25 | 90<br>6/7 | 100<br>3/8 | 3/4 | 70 90<br>5/23 | 80<br>5/8 | 90<br>9/0 | 55 70<br>80 90 |
| 5 | 100<br>2/29 | 100<br>6/34 | 70 100<br>12/39 | 100<br>17/30 | 90<br>14/24 | 80<br>14/39 | 14/40 | 80<br>16/38 | 100<br>18/34 | 90<br>13/3 | 80<br>100 |
| 10 | 100<br>5/55 | 100<br>13/65 | 90<br>9/66 | 90<br>8/68 | 70 80<br>11/67 | 100<br>11/59 | 3/71 | 100<br>9/65 | 100<br>7/49 | 80<br>11/49 | 85<br>100 |
| 50 | 100<br>5/0 | 100<br>4/0 | 90<br>8/0 | 90<br>7/0 | 100<br>2/0 | 80<br>8/0 | 1/3 | 60 80<br>11/1 | 100<br>7/1 | 100<br>14/5 | 95 |

PERCENTAGE AND RATE DATA FOR NORMALLY ACHIEVING SUBJECTS IN CLASSROOM 5--CONTINUED

| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
|---|---|---|---|---|---|---|---|---|---|---|
| 100<br>13/1 | 100<br>16/1 | 100<br>20/0 | 80<br>9/0 | 80<br>20/0 | 90<br>19/3 | 80<br>7/0 | 100<br>18/0 | 90<br>22/0 | 100<br>12/0 | 100<br>22/0 |
| 100<br>12/2 | 90<br>10/2 | 10/1 | 70<br>7/0 | 70<br>8/0 | 60<br>15/1 | 70<br>5/2 | 60<br>8/0 | 80<br>12/1 | 90<br>1/1 | 100<br>13/1 |
| 100<br>15/8 | 80<br>6/5 | 50<br>8/14 | 60<br>12/11 | 80<br>11/2 | 50<br>18/2 | 80<br>11/6 | 60<br>14/5 | 60 | 40 |  |
| 90<br>14/1 | 90 | 70<br>15/3 | 90<br>4/0 | 80<br>5/0 | 50<br>14/0 | 80<br>6/1 | 90<br>2/0 | 90<br>9/0 | 80<br>13/1 | 90<br>18/2 |
| 90<br>10/32 | 90<br>2/0 | 80<br>6/0 | 60<br>7/3 | 8/0 | 90<br>10/0 | 40<br>7/0 | 7/1 | 14/0 | 40<br>7/1 | 90<br>14/0 |
| 90<br>12/3 | 100<br>19/21 | 80<br>15/11 | 90<br>7/6 | 80<br>18/0 | 60<br>16/2 | 60 | 90 | 20/0 | 13/1 | 100<br>18/2 |
| 90<br>22/11 | 90 | 100<br>20/2 | 100<br>15/2 | 100<br>27/0 | 90<br>27/2 | 100<br>8/1 | 100<br>20/0 | 100<br>39/1 | 100<br>46/0 | 100<br>53/2 |
| 80<br>15/17 | 90<br>15/18 | 70<br>13/12 | 60<br>15/6 | 90<br>10/4 | 60<br>16/0 | 70<br>14/11 | 90<br>17/10 | 100<br>18/2 | 100<br>21/10 | 100<br>17/9 |
| 100<br>15/0 | 100<br>11/12 | 90<br>13/0 | 90<br>7/0 | 80<br>18/0 | 90<br>17/1 | 90<br>9/6 | 100<br>6/1 | 70<br>24/0 | 100<br>15/4 | 100<br>24/2 |
| 12/3 | 90<br>10/9 | 60<br>12/7 | 50<br>6/7 | 60<br>11/3 | 80<br>16/2 | 50<br>8/3 | 50<br>14/4 | 100<br>12/2 | 70<br>14/5 | 100<br>13/3 |
| 80<br>18/11 | 90<br>18/10 | 100<br>16/6 | 80<br>18/5 | 80<br>16/0 | 90 | 10/1 | 70<br>11/1 | 90<br>22/0 | 100<br>12/0 | 100<br>22/0 |
| 90<br>23/4 | 100<br>16/7 | 100 | 70<br>19/2 | 90<br>24/2 | 80<br>24/0 | 100<br>8/3 | 100<br>18/0 | 100<br>36/2 | 90<br>44/4 | 100<br>49/2 |
| 80<br>13/6 | 100<br>7/11 | 70<br>16/0 | 100<br>5/0 | 60<br>9/1 | 60<br>14/0 | 80<br>8/3 | 60<br>8/2 | 9/0 | 80<br>13/4 | 70<br>15/1 |
| 90<br>16/15 | 90<br>16/16 | 100<br>18/15 | 80<br>11/10 | 80<br>15/10 | 90<br>14/1 | 100<br>13/8 | 90<br>14/6 | 80<br>16/3 | 90<br>24/10 | 100<br>15/4 |
| 100<br>7/4 | 90<br>5/2 | 50<br>2/1 | 60<br>5/0 | 60<br>6/3 | 50<br>6/4 | 60<br>2/8 | 60<br>6/2 | 50<br>8/1 | 50<br>11/4 | 100<br>7/2 |
| 100<br>16/5 | 80<br>19/11 | 80<br>21/1 | 70<br>14/6 | 90<br>14/2 | 90<br>17/1 | 80<br>7/0 | 100<br>13/1 | 80<br>17/1 | 100<br>2/0 | 100<br>23/1 |
| 100<br>8/13 | 100<br>11/28 | 80<br>6/14 | 80<br>22/21 | 60<br>16/0 | 80<br>16/0 | 90<br>11/4 | 30<br>18/2 | 80<br>14/0 | 100<br>22/5 | 100<br>16/2 |
| 12/0 | 90<br>11/1 | 70<br>18/0 | 100<br>10/0 |  | 18/0 | 70<br>7/0 | 60<br>14/0 | 100<br>22/0 | 19/5 | 90<br>24/2 |

Note: Percentage scores are recorded over rate (correct digits/incorrect digits) data.

## PERCENTAGE AND RATE DATA FOR NORMALLY ACHIEVING SUBJECTS IN CLASSROOM 6

| PRE | C1 | C2 | C3 | R4 | R5 | R6 | MN7 | A8 | A9 | A10 | POST |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 10 100<br>1/53 | 100<br>0/7 | 100<br>7/1 | 100<br>0/1 | 70 100<br>6/6 | 100<br>7/0 | 3/0 | 90<br>9/3 | 90<br>11/3 | 80<br>9/0 | 85<br>100 |
| 15 | 100<br>3/51 | 100<br>14/41 | 100<br>7/14 | 100<br>19/31 | 100<br>9/28 | 100<br>7/47 | 3/3 | 90<br>11/51 | 100<br>22/41 | 100<br>7/11 | 90<br>100 |
| 5 | 60 100<br>4/12 | 100<br>6/8 | 100<br>10/7 | 100<br>9/14 | 90<br>9/22 | 80<br>7/14 | 5/0 | 80<br>7/0 | 100<br>8/0 | 100<br>8/0 | 85<br>100 |
| 25 | 80<br>3/51 | 100<br>12/42 | 100<br>15/30 | 100<br>16/39 | 100<br>12/42 | 80<br>9/45 | 7/31 | 100<br>13/42 | 90<br>7/1 | 100 | 85<br>100 |
| 5 | 90<br>3/54 | 100<br>6/48 | 100<br>12/45 | 100<br>21/36 | 90<br>13/32 | 100<br>17/17 | 5/2 | 70 100<br>12/0 | 90<br>5/7 | 100<br>8/0 | 65<br>90 |
| 0 | 60 100<br>2/52 | 100<br>3/51 | 60<br>3/55 | 100<br>1/17 | 100<br>3/7 | 90<br>3/32 | 2/52 | 4/8 | 80 100<br>6/1 | 70<br>3/0 | 70 |
| 5 | 90<br>7/32 | 100<br>8/47 | 100<br>8/0 | 100<br>5/3 | 100<br>11/1 | 100<br>5/0 | 7/0 | 70<br>12/0 | 100<br>3/1 | 90<br>9/0 | 95 |
| 10 | 90<br>2/52 | 100<br>8/0 | 100<br>10/0 | 100<br>9/1 | 100<br>4/1 | 80<br>8/0 | 5/0 | 90<br>2/2 | 90<br>8/0 | 100<br>9/0 | 70<br>90 |
| 0 | 50 100<br>0/54 | 100<br>4/51 | 40 70<br>8/46 | 100<br>7/47 | 100<br>6/16 | 70 80<br>6/30 | 5/0 | 70<br>9/37 | 60 90<br>14/40 | 100<br>3/6 | 45 70<br>90 |
| 15 | 70 100<br>10/28 | 100<br>8/23 | 100<br>17/49 | 100<br>25/38 | 100<br>13/31 | 90<br>12/20 | 9/6 | 90<br>12/0 | 100<br>9/5 | 100<br>9/0 | 85<br>100 |
| 10 | 70 100<br>2/52 | 100<br>2/39 | 100<br>7/23 | 100<br>6/12 | 100<br>3/8 | 90<br>7/0 | 5/0 | 80<br>11/0 | 80<br>8/0 | 100<br>4/0 | 80 |
| 0 | 90<br>8/53 | 100<br>10/0 | 100<br>8/2 | 100<br>0/3 | 100<br>7/0 | 100<br>8/0 | 5/0 |  | 100<br>15/0 | 90<br>9/0 | 100 |
| 5 | 70 100<br>3/27 | 100<br>6/34 | 100<br>10/29 | 100<br>3/10 | 100<br>11/6 | 90<br>8/2 | 5/0 | 100<br>6/1 | 100<br>10/0 | 90<br>9/0 | 80<br>95 |

PERCENTAGE AND RATE DATA FOR NORMALLY ACHIEVING SUBJECTS IN CLASSROOM 6--CONTINUED

| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
|---|---|---|---|---|---|---|---|---|---|---|
| 100<br>12/0 | 100<br>9/1 | 80<br>11/4 | 80<br>7/0 | 60<br>10/2 | 60<br>12/0 | 90<br>11/0 | 60<br>2/0 | 100<br>4/0 | 80<br>2/0 | 90<br>4/0 |
| 100<br>12/0 | 100<br>18/0 | 100<br>22/4 | 9/0 | 70<br>17/1 | 60<br>24/4 | 100<br>10/3 | 70<br>15/3 | 70<br>19/3 | 90<br>17/0 | 90<br>14/2 |
| 2/0 | 90<br>8/0 | 80<br>9/0 | 50<br>5/0 | 40<br>8/0 | 14/0 | 70<br>13/0 | 80<br>10/0 | 80<br>24/0 | 100<br>23/0 | 80<br>20/0 |
| 80<br>2/0 | 70<br>12/2 | 60<br>11/0 | 70<br>3/2 | 70<br>9/1 | 70<br>8/0 | 80<br>12/1 | 70<br>9/1 | 80<br>9/2 | 60<br>18/5 | 90<br>13/3 |
| 100<br>10/0 | 90<br>10/5 | 80<br>9/2 | 50<br>3/2 | 80<br>7/3 | 60<br>10/0 | 90<br>16/1 | 90<br>13/2 | 90<br>24/0 | 100<br>21/0 | 100<br>14/0 |
| 90<br>2/0 | 100<br>2/0 | 80<br>7/2 | 3/0 | 50<br>0/14 | 1/1 | 60<br>6/1 | 60<br>3/0 | 4/0 | 2/0 | 4/0 |
| 80<br>10/2 | 100<br>7/0 | 80<br>20/2 | 70<br>7/0 | 100<br>11/3 | 80<br>14/0 | 90<br>15/0 | 80<br>14/1 | 90<br>23/1 | 80<br>25/2 | 90<br>24/0 |
| 80 | 90<br>2/0 | 100<br>6/2 | 90<br>9/0 | 90<br>5/0 | 60<br>7/1 | 100<br>7/2 | 100<br>8/0 | 100<br>13/1 | 70<br>14/0 | 90<br>14/0 |
| 90<br>6/12 | 90<br>10/16 | 60<br>9/1 | 80<br>5/0 | 40<br>6/10 | 20<br>1/1 | 30<br>2/8 | 70<br>8/6 | 90 | 40 | 20 |
| 100<br>4/1 | 100<br>6/1 | 100<br>9/0 | 80<br>5/0 | 70<br>8/0 | 80<br>7/1 | 100<br>9/0 | 100<br>7/1 | 90<br>9/0 | 90<br>8/0 | 100<br>18/0 |
| 80<br>2/0 | 90<br>5/0 | 80<br>9/0 | 70<br>5/0 | 40<br>7/0 | 50<br>3/0 | 70<br>3/0 | 80<br>9/1 | 70<br>8/0 | 80<br>7/1 | 100<br>6/3 |
| 100<br>12/0 | 80<br>15/2 | 100<br>20/0 | 80 | 90<br>16/0 | 90<br>18/0 | 90<br>7/0 | 100<br>14/0 | 22/6 | 90<br>17/0 | 90<br>23/5 |
| 90<br>8/0 | 80<br>11/4 | 90<br>11/0 | 70<br>7/0 | 80<br>8/0 | 50<br>8/0 | 60<br>7/0 | 90<br>13/2 | 80<br>15/3 | 90<br>17/2 | 80<br>16/0 |

Note: Percentage scores are recorded over rate (correct digits/incorrect digits) data.

# APPENDIX L

## INDIVIDUAL AND SUMMARY DATA ON MEASURES OF ACCURACY AND FLUENCY FOR CLASSROOMS IN STUDY I

DATA FOR UNTIMED INDEPENDENT PRACTICE IN CLASSROOM 1
(percentage correct)

| Normally Achieving Subjects | | | Exceptional Subjects | | |
|---|---|---|---|---|---|
| | | | 1.1 | 1.2 | 1.3 |
| Pretest | 0-40 | 12.5 (16) | 15 | 10 | 10 |
| Lesson | Range (%-%) | Median (%) | Score (%) | Score (%) | Score (%) |
| 1 | 50-100 | 90 (16) | $\underline{100}$ √ | $\underline{90}$ √ | $\underline{90}$ √ |
| 2 | 60-100 | 90 (16) | $\underline{90}$ √ | $\underline{90}$ √ | 60 √ |
| 3 | 40-100 | 90 (16) | 80 √ | $\underline{100}$ √ | $\underline{100}$ √ |
| 4 | 80-100 | 100 (16) | 90 √ | $\underline{100}$ √ | $\underline{100}$ √ |
| 5 | 70-100 | 90 (15) | $\underline{100}$ √ | $\underline{90}$ √ | $\underline{90}$ √ |
| 6 | 70-100 | 100 (14) | 80 √ | $\underline{100}$ √ | 90 √ |
| 8 | 70-100 | 90 (16) | $\underline{90}$ √ | $\underline{100}$ √ | 80 √ |
| 9 | 80-100 | 100 (16) | 90 √ | $\underline{100}$ √ | $\underline{100}$ √ |
| 10 | 90-100 | 90 (15) | 80 | $\underline{90}$ √ | $\underline{100}$ √ |
| Posttest | 50-95 | 80 (15) | $\underline{90}$ √ | 70 √ | 65 √ |
| 11 | 60-100 | 100 (14) | | $\underline{100}$ √ | $\underline{100}$ √ |
| 12 | 70-100 | 100 (15) | | $\underline{100}$ √ | $\underline{100}$ √ |
| 13 | 60-100 | 80 (16) | 70 √ | 60 √ | $\underline{80}$ √ |
| 14 | 40-100 | 70 (14) | 40 √ | 40 √ | $\underline{90}$ √ |
| 15 | 50-100 | 85 (14) | 50 √ | 70 √ | 70 √ |
| 16 | 40-100 | 75 (14) | | 70 √ | 50 √ |
| 17 | 60-100 | 90 (14) | 70 √ | 80 √ | 70 √ |
| 18 | 40-100 | 70 (12) | 50 √ | 60 √ | 40 √ |
| 19 | 50-100 | 90 (16) | | 60 √ | 60 √ |
| 20 | 60-90 | 80 (16) | | $\underline{80}$ √ | 70 √ |
| 21 | 50-100 | 100 (14) | | $\underline{100}$ √ | |
| % of lessons-within range | | | 93% | 100% | 100% |
| % of lessons-median+ | | | 33% | 62% | 50% |

√ = within range ____ = score at or above group median

Note: No learning sheet was required for lesson 7.
Number in parentheses is the number of subjects reported for range and median.

DATA FOR RATE IN CLASSROOM 1
(number of correct digits per minute)

| Normally Achieving Subjects | | | Exceptional Subjects | | |
|---|---|---|---|---|---|
| Digit Writing Rate | 30-60 | 47.5 (16) | 1.1<br>41 | 1.2<br>42 | 1.3<br>38 |
| Measure | Range | Median # | # | # | # |
| 1 | 2-8 | 3 (15) | 3 √ | 1 | 3 √ |
| 2 | 1-12 | 4 (14) | 4 √ | 4 √ | 5 √ |
| 3 | 2-14 | 8 (12) | 11 √ | 5 √ | 3 √ |
| 4 | 0-17 | 5 (13) | 13 √ | 7 √ | 2 √ |
| 5 | 1-12 | 7 (14) | 14 √* | 7 √ | 2 √ |
| 6 | 4-16 | 9 (14) | 10 √ | 8 √ | 6 √ |
| 7 | 3-15 | 6.5(16) | 3 √ | 4 √ | 7 √ |
| 8 | 3-18 | 10 (16) | 3 √ | 6 √ | 13 √ |
| 9 | 5-21 | 14 (15) | 3 | | 15 √ |
| 10 | 4-22 | 16 (15) | 13 √ | 5 √ | 12 √ |
| 11 | 7-17 | 12 (14) | 10 √ | 1 | 11 √ |
| 12 | 2-18 | 12 (16) | 8 √ | 14 √ | 16 √ |
| 13 | 7-22 | 15 (15) | 6 | 11 √ | 13 √ |
| 14 | 12-31 | 18 (11) | 8 | 13 √ | 17 √ |
| 15 | 5-15 | 8 (15) | 6 √ | 9 √ | 7 √ |
| 16 | 8-31 | 16 (15) | 8 √ | 8 √ | 14 √ |
| 17 | 14-27 | 18 (13) | 14 √ | 16 √ | 18 √ |
| 18 | 7-24 | 13 (13) | 7 √ | 16 √ | 13 √ |
| 19 | 8-28 | 20 (16) | 14 √ | 11 √ | 10 √ |
| 20 | 8-24 | 16 (16) | | | 16 √ |
| 21 | | | | | |
| % of measures-within range | | | 84% | 89% | 100% |
| % of measures-median+ | | | 31% | 33% | 45% |

√ = within range ____ = score at or above group median
* = beyond upper limit of range
Note: Number in parentheses is the number of subjects reported for range and median.

DATA FOR UNTIMED INDEPENDENT PRACTICE IN CLASSROOM 2
(percentage correct)

| Normally Achieving Subjects | | | Exceptional Subjects | |
|---|---|---|---|---|
| | | | 2.1 | 2.2 |
| Pretest | 5-45 | 15 (17) | 30 | 5 |
| Lesson | Range (%-%) | Median (%) | Score (%) | Score (%) |
| 1 | 80-100 | 100 (17) | 100 √ | 90 √ |
| 2 | 80-100 | 100 (17) | 100 √ | 100 √ |
| 3 | 80-100 | 90 (17) | 80 √ | 80 √ |
| 4 | 80-100 | 100 (17) | 100 √ | 80 √ |
| 5 | 60-100 | 100 (17) | 90 √ | 70 √ |
| 6 | 80-100 | 90 (17) | 100 √ | 100 √ |
| 8 | 70-100 | 90 (17) | 90 √ | 80 √ |
| 9 | 50-100 | 100 (17) | 80 √ | 80 √ |
| 10 | 80-100 | 100 (17) | 100 √ | 90 √ |
| Posttest | 60-90 | 75 (17) | 65 √ | 30 |
| 11 | 70-100 | 90 (17) | 100 √ | 90 √ |
| 12 | 80-100 | 100 (16) | 80 √ | 80 √ |
| 13 | 40-100 | 95 (16) | 60 √ | 50 √ |
| 14 | 50-100 | 80 (15) | 100 √ | 50 √ |
| 15 | 50-100 | 75 (16) | 40 | |
| 16 | 40-100 | 90 (15) | 40 √ | |
| 17 | 60-100 | 90 (17) | 90 √ | 70 √ |
| 18 | 50-100 | 70 (15) | 90 √ | 50 √ |
| 19 | 70-100 | 90 (15) | 80 √ | 60 |
| 20 | 70-100 | 95 (16) | 60 | 60 |
| 21 | 80-100 | 90 (17) | 90 √ | 80 √ |
| % of lessons-within range | | | 90% | 84% |
| % of lessons-median+ | | | 52% | 16% |

√ = within range ____ = score at or above group median

Note: No learning sheet was required for lesson 7. Number in parentheses is the number of subjects reported for range and median.

DATA FOR RATE IN CLASSROOM 2
(number of correct digits per minute)

| Normally Achieving Subjects | | | Exceptional Subjects | |
|---|---|---|---|---|
| Digit Writing Rate | 47-84 | 60 (17) | 2.1<br>40 | 2.2<br>43 |
| Measure | Range | Median # | # | # |
| 1 | 2-14 | 4 (16) | 4 √ | 2 √ |
| 2 | 1-17 | 8 (16) | 2 √ | 5 √ |
| 3 | 0-16 | 8 (17) | 9 √ | 4 √ |
| 4 | 1-16 | 6 (17) | 5 √ | 4 √ |
| 5 | 0-17 | 8 (17) | 9 √ | 4 √ |
| 6 | 3-18 | 8 (17) | 8 √ | 3 √ |
| 7 | 2-9 | 5 (17) | 7 √ | 0 |
| 8 | 0-15 | 10 (17) | 12 √ | 4 √ |
| 9 | 2-17 | 7 (17) | 5 √ | 5 √ |
| 10 | 6-17 | 9 (16) | 9 √ | 6 √ |
| 11 | 2-18 | 12 (15) | 10 √ | 1 |
| 12 | 0-18 | 10 (17) | 8 √ | 3 √ |
| 13 | 3-22 | 13 (17) | 11 √ | 5 √ |
| 14 | 2-8 | 6 (15) | 6 √ | 3 √ |
| 15 | 3-24 | 14 (16) | 8 √ | |
| 16 | 7-31 | 16 (15) | 15 √ | |
| 17 | 3-11 | 6 (17) | 5 √ | 4 √ |
| 18 | 2-19 | 10 (16) | 10 √ | 5 √ |
| 19 | 9-22 | 16 (16) | 14 √ | 5 |
| 20 | 9-22 | 12.5 (16) | 11 √ | 9 √ |
| 21 | 7-25 | 15 (17) | 13 √ | 12 √ |
| % of measures-within range | | | 100% | 84% |
| % of measures-median+ | | | 43% | 0% |

√ = within range ____ = score at or above group median

*Note*: Number in parentheses is the number of subjects reported for range and median.

DATA FOR UNTIMED INDEPENDENT PRACTICE IN CLASSROOM 3
(percentage correct)

| Normally Achieving Subjects | | | Exceptional Subject |
|---|---|---|---|
| | | | 3.1 |
| Pretest | 5-70 | 15 (19) | 50 |
| Lesson | Range (%-%) | Median (%) | Score (%) |
| 1 | 10-100 | 90 (19) | 70 √ |
| 2 | 80-100 | 100 (19) | 100 √ |
| 3 | 50-100 | 90 (19) | 80 √ |
| 4 | 80-100 | 100 (18) | 100 √ |
| 5 | 60-100 | 85 (18) | 100 √ |
| 6 | 40-100 | 100 (18) | 100 √ |
| 8 | 60-100 | 90 (18) | 100 √ |
| 9 | 80-100 | 100 (18) | 100 √ |
| 10 | 70-100 | 90 (17) | 100 √ |
| Posttest | 45-100 | 80 (19) | 80 √ |
| 11 | 80-100 | 90 (19) | 80 √ |
| 12 | 60-100 | 100 (19) | 90 √ |
| 13 | 50-100 | 80 (19) | 70 √ |
| 14 | 40-100 | 70 (18) | 80 √ |
| 15 | 30-100 | 70 (19) | 70 √ |
| 16 | 40-100 | 70 (19) | 60 √ |
| 17 | 50-100 | 80 (19) | 80 √ |
| 18 | 30-100 | 70 (19) | 90 √ |
| 19 | 60-90 | 80 (19) | 90 √ |
| 20 | 60-100 | 90 (18) | 80 √ |
| 21 | 60-100 | 90 (18) | 50 |
| % of lessons-within range | | | 95% |
| % of lessons-median+ | | | 62% |

√ = within range ____ = score at or above group median

Note: No learning sheet was required for lesson 7. Number in parentheses is the number of subjects reported for range and median.

DATA FOR RATE IN CLASSROOM 3
(number of correct digits per minute)

| Normally Achieving Subjects | | | Exceptional Subject |
|---|---|---|---|
| Digit Writing Rate | 30-70 | 50 (19) | 3.1<br>51 |
| Measure | Range | Median # | # |
| 1 | 1-10 | 4.5(16) | 7 √ |
| 2 | 3-12 | 9 (15) | 14 √* |
| 3 | 0-11 | 6 (15) | 12 √* |
| 4 | 0-14 | 3.5(16) | 11 √ |
| 5 | 0-15 | 10 (16) | 17 √* |
| 6 | 3-14 | 7.5(18) | 16 √* |
| 7 | 2-8 | 5 (17) | 6 √ |
| 8 | 1-19 | 9.5(18) | 11 √ |
| 9 | 2-17 | 10 (18) | 9 √ |
| 10 | 5-16 | 12 (16) | 19 √* |
| 11 | 4-15 | 12 (19) | 11 √ |
| 12 | 2-16 | 8 (19) | 8 √ |
| 13 | 8-22 | 14 (19) | 22 √ |
| 14 | 4-11 | 7 (17) | 14 √* |
| 15 | 5-24 | 14 (19) | 18 √ |
| 16 | 10-26 | 17 (19) | 31 √* |
| 17 | 4-26 | 10 (19) | 12 √ |
| 18 | 0-20 | 16 (19) | 19 √ |
| 19 | 7-28 | 18 (19) | 22 √ |
| 20 | 7-29 | 12 (17) | 24 √ |
| 21 | 8-34 | 14 (18) | 19 √ |

| | |
|---|---|
| % of measures-within range | 100% |
| % of measures-median+ | 90% |

√ = within range ____ = score at or above group median
* = beyond upper limit of range
Note: Number in parentheses is the number of subjects reported for range and median.

# APPENDIX M
# INDIVIDUAL AND SUMMARY DATA ON MEASURES OF ACCURACY AND FLUENCY FOR CLASSROOMS IN STUDY II

DATA FOR UNTIMED INDEPENDENT PRACTICE IN CLASSROOM 4
(percentage correct)

| Normally Achieving Subjects | | | Exceptional Subjects | | |
|---|---|---|---|---|---|
| Pretest | 0-50 | 20 (16) | 4.1<br>15 | 4.2<br>20 | 4.3<br>15 |
| Lesson | Range (%-%) | Median (%) | Score (%) | Score (%) | Score (%) |
| 1 | 50-100 | 100 (16) | 40 | 90 √ | 70 √ |
| 2 | 70-100 | 90 (16) | 90 √ | 90 √ | 80 √ |
| 3 | 70-100 | 90 (16) | 60 | 100 √ | 80 √ |
| 4 | 90-100 | 100 (16) | 100 √ | 100 √ | 90 √ |
| 5 | 70-100 | 90 (15) | 90 √ | 90 √ | 80 √ |
| 6 | 60-100 | 90 (16) | 90 √ | 90 √ | 90 √ |
| 8 | 70-100 | 90 (16) | 100 √ | 90 √ | 100 √ |
| 9 | 50-100 | 100 (16) | 80 √ | 100 √ | 90 √ |
| 10 | 70-100 | 100 (16) | 90 √ | 100 √ | 90 √ |
| Posttest | 65-100 | 90 (16) | 100 √ | 100 √ | 95 √ |
| 11 | 70-100 | 100 (16) | 100 √ | 100 √ | 100 √ |
| 12 | 80-100 | 100 (15) | 90 √ | 100 √ | 100 √ |
| 13 | 50-100 | 70 (14) | 70 √ | 70 √ | 70 √ |
| 14 | 40-100 | 75 (16) | 90 √ | 70 √ | 60 √ |
| 15 | 90-100 | 100 (14) | | 100 √ | 100 √ |
| 16 | 80-100 | 100 (16) | 80 √ | 100 √ | 40 |
| 17 | 80-100 | 100 (16) | 70 | 100 √ | |
| 18 | 60-100 | 90 (15) | | | |
| 19 | 90-100 | 100 (15) | 70 | 90 √ | 80 |
| 20 | 80-100 | 100 (15) | 80 √ | | 80 √ |
| 21 | 60-100 | 85 (14) | 80 √ | 100 √ | 80 √ |
| % of lessons-within range | | | 79% | 100% | 89% |
| % of lessons-median+ | | | 47% | 84% | 37% |

√ = within range ____ = score at or above group median
Note: No learning sheet was required for lesson 7.
Number in parentheses is the number of subjects reported for range and median.

DATA FOR RATE IN CLASSROOM 4
(number of correct digits per minute)

| Normally Achieving Subjects | | | Exceptional Subjects | | |
|---|---|---|---|---|---|
| Digit Writing Rate | 30-100 | 53.5 (16) | 4.1<br>56 | 4.2<br>50 | 4.3<br>64 |
| Measure | Range | Median # | # | # | # |
| 1 | 1-9 | 5 (15) | 2 √ | 3 √ | 3 √ |
| 2 | 4-17 | 8 (13) | 4 √ | 5 √ | 2 |
| 3 | 2-14 | 6 (15) | 9 √ | 6 √ | 4 √ |
| 4 | 2-20 | 11 (15) | 8 √ | 14 √ | 9 √ |
| 5 | 6-16 | 9 (15) | 13 √ | 8 √ | 3 |
| 6 | 7-17 | 10 (16) | 8 √ | 13 √ | |
| 7 | 2-20 | 7 (15) | 9 √ | 19 √ | 5 √ |
| 8 | • | • | • | • | • |
| 9 | 4-19 | 12.5 (16) | 8 √ | 12 √ | 10 √ |
| 10 | 9-29 | 13 (14) | 10 √ | 21 √ | |
| 11 | 2-16 | 11 (15) | 15 √ | 8 √ | 2 √ |
| 12 | 5-16 | 10 (15) | 16 √ | 16 √ | 1 |
| 13 | 8-38 | 13 (15) | 13 √ | 26 √ | 9 √ |
| 14 | 3-38 | 9.5 (14) | 8 √ | 10 √ | 5 √ |
| 15 | 7-31 | 18 (14) | 13 √ | 19 √ | 14 √ |
| 16 | 7-33 | 17 (16) | 12 √ | 24 √ | 14 √ |
| 17 | 5-18 | 9 (16) | | 13 √ | |
| 18 | 2-20 | 14.5 (16) | | 14 √ | |
| 19 | 7-37 | 17 (14) | | 21 √ | 16 √ |
| 20 | 7-25 | 17.5 (14) | | 18 √ | 12 √ |
| 21 | 8-24 | 20.5 (14) | | 19 √ | 22 √ |
| % of measures-within range | | | 100% | 100% | 81% |
| % of measures-median+ | | | 40% | 65% | 6% |

√ = within range ____ = score at or above group median
• = probe not given
Note: Number in parentheses is the number of subjects reported for range and median.

DATA FOR UNTIMED INDEPENDENT PRACTICE IN CLASSROOM 5
(percentage correct)

| Normally Achieving Subjects | | | Exceptional Subject |
|---|---|---|---|
| | | | 5.1 |
| Pretest | 0-50 | 10 (18) | 5 |
| Lesson | Range (%-%) | Median (%) | Score (%) |
| 1 | 80-100 | 100 (18) | 100 √ |
| 2 | 90-100 | 100 (18) | 100 √ |
| 3 | 60-100 | 90 (18) | 100 √ |
| 4 | 90-100 | 100 (18) | 80 |
| 5 | 70-100 | 95 (18) | 80 √ |
| 6 | 70-100 | 95 (18) | 70 √ |
| 8 | 70-100 | 85 (18) | 60 |
| 9 | 80-100 | 100 (16) | 90 √ |
| 10 | 70-100 | 90 (17) | 90 √ |
| Posttest | 55-95 | 87.5 (18) | 60 √ |
| 11 | 80-100 | 90 (16) | 60 |
| 12 | 80-100 | 90 (18) | 90 √ |
| 13 | 50-100 | 80 (17) | 80 √ |
| 14 | 50-100 | 80 (18) | |
| 15 | 60-100 | 80 (16) | 60 √ |
| 16 | 50-90 | 80 (17) | 30 |
| 17 | 40-100 | 80 (17) | 60 √ |
| 18 | 50-100 | 90 (17) | 30 |
| 19 | 50-100 | 90 (16) | 90 √ |
| 20 | 40-100 | 90 (16) | 60 √ |
| 21 | 70-100 | 100 (17) | 100 √ |
| % of lessons-within range | | | 75% |
| % of lessons-median+ | | | 40% |

√ = within range ____ = score at or above group median

Note: No learning sheet was required for lesson 7.
Number in parentheses is the number of subjects reported for range and median.

DATA FOR RATE IN CLASSROOM 5
(number of correct digits per minute)

| Normally Achieving Subjects | | | Exceptional Subject |
|---|---|---|---|
| Digit Writing Rate | 38-88 | 51.5 (18) | 5.1<br>80 |
| Measure | Range | Median # | # |
| 1 | 0-7 | 4 (17) | 2 √ |
| 2 | 2-21 | 6 (16) | 4 √ |
| 3 | 3-22 | 8.5(16) | 9 √ |
| 4 | 1-17 | 6.5(18) | |
| 5 | 2-22 | 9 (18) | |
| 6 | 3-21 | 8.5(18) | |
| 7 | 1-17 | 7 (18) | |
| 8 | 1-24 | 11 (18) | |
| 9 | 4-18 | 11.5(16) | |
| 10 | 6-21 | 13 (17) | 11 √ |
| 11 | 7-23 | 13.5(18) | 17 √ |
| 12 | 2-19 | 11 (16) | 9 √ |
| 13 | 2-21 | 15 (17) | |
| 14 | 4-22 | 9.5(18) | 9 √ |
| 15 | 5-27 | 14 (17) | 8 √ |
| 16 | 6-27 | 16 (17) | 16 √ |
| 17 | 5-14 | 8 (17) | 10 √ |
| 18 | 2-20 | 14 (17) | 4 √ |
| 19 | 8-39 | 17 (17) | 14 √ |
| 20 | 1-46 | 13 (17) | 15 √ |
| 21 | 7-53 | 18 (17) | 16 √ |

| | |
|---|---|
| % of lessons-within range | 100% |
| % of lessons-median+ | 36% |

√ = within range ____ = score at or above group median

*Note*: Number in parentheses is the number of subjects reported for range and median.

DATA FOR UNTIMED INDEPENDENT PRACTICE IN CLASSROOM 6
(percentage correct)

| Normally Achieving Subjects | | | Exceptional Subjects | | |
|---|---|---|---|---|---|
| | | | 6.1 | 6.2 | 6.3 |
| Pretest | 0-25 | 5 (13) | 20 | 0 | 5 |
| Lesson | Range (%-%) | Median (%) | Score (%) | Score (%) | Score (%) |
| 1 | 10-100 | 70 (13) | 100 √ | 10 √ | 50 √ |
| 2 | 100 | 100 (13) | 100 √ | 100 √ | 100 √ |
| 3 | 40-100 | 100 (13) | 100 √ | 100 √ | 90 √ |
| 4 | 100 | 100 (13) | 100 √ | 100 √ | 100 √ |
| 5 | 70-100 | 100 (13) | 100 √ | 100 √ | 100 √ |
| 6 | 70-100 | 90 (13) | 90 √ | 80 √ | 80 √ |
| 8 | 70-100 | 90 (11) | 70 √ | | 90 √ |
| 9 | 60-100 | 90 (13) | 100 √ | 80 √ | 90 √ |
| 10 | 70-100 | 100 (13) | 100 √ | 90 √ | 90 √ |
| Posttest | 45-100 | 85 (13) | 70 √ | 70 √ | 65 √ |
| 11 | 80-100 | 90 (12) | 80 √ | 60 | |
| 12 | 70-100 | 90 (13) | 80 √ | | 90 √ |
| 13 | 60-100 | 80 (13) | 90 √ | 90 √ | 80 √ |
| 14 | 50-90 | 70 (11) | 100 √ | 50 √ | |
| 15 | 40-100 | 70 (13) | 100 √ | 80 √ | |
| 16 | 20-90 | 60 (11) | 90 √ | 80 √ | |
| 17 | 30-100 | 90 (13) | 90 √ | 90 √ | |
| 18 | 60-100 | 80 (13) | 90 √ | 80 √ | 100 √ |
| 19 | 70-100 | 90 (11) | 80 √ | 70 √ | |
| 20 | 40-100 | 85 (12) | 80 √ | 70 √ | 70 √ |
| 21 | 20-100 | 90 (12) | | | 70 √ |
| % of lessons-within range | | | 100% | 94% | 100% |
| % of lessons-median+ | | | 70% | 50% | 53% |

√ = within range ____ = score at or above group median

Note: No learning sheet was required for lesson 7.
Number in parentheses is the number of subjects reported for range and median.

DATA FOR RATE IN CLASSROOM 6
(number of correct digits per minute)

| Normally Achieving Subjects | | | Exceptional Subjects | | |
|---|---|---|---|---|---|
| Digit Writing Rate | 32-100 | 62 (13) | 6.1<br>43 | 6.2<br>80 | 6.3<br>50 |
| Measure | Range | Median # | # | # | # |
| 1 | 0-10 | 3 (13) | 2 √ | 5 √ | 5 √ |
| 2 | 0-14 | 6 (13) | 14 √ | 1 √ | 5 √ |
| 3 | 3-17 | 8 (13) | 14 √ | 5 √ | 11 √ |
| 4 | 0-25 | 7 (13) | 17 √ | 6 √ | 5 √ |
| 5 | 3-13 | 9 (13) | 15 √* | 6 √ | 12 √ |
| 6 | 3-17 | 7 (13) | 21 √* | 10 √ | 8 √ |
| 7 | 2-9 | 5 (13) | 11 √* | 3 √ | 8 √ |
| 8 | 2-13 | 11 (12) | 17 √* | | 19 √* |
| 9 | 3-22 | 8 (13) | 15 √ | 15 √ | 21 √ |
| 10 | 3-9 | 8.5 (12) | 12 √* | 3 √ | 15 √* |
| 11 | 2-12 | 7 (12) | 10 √ | | 5 √ |
| 12 | 2-18 | 9 (13) | 13 √ | 16 √ | 8 √ |
| 13 | 6-22 | 9 (13) | 23 √* | 12 √ | 10 √ |
| 14 | 3-9 | 5 (12) | 5 √ | 7 √ | 3 √ |
| 15 | 0-17 | 8 (13) | 17 √ | 8 √ | 13 √ |
| 16 | 1-24 | 8 (13) | 16 √ | 12 √ | |
| 17 | 2-16 | 9 (13) | 16 √ | 10 √ | 9 √ |
| 18 | 2-15 | 9 (13) | 20 √* | | 12 √ |
| 19 | 4-24 | 14 (12) | 24 √ | | 17 √ |
| 20 | 2-25 | 17 (12) | 24 √ | | 18 √ |
| 21 | 4-24 | 14 (12) | 26 √* | | 18 √ |
| % of measures-within range | | | 100% | 100% | 100% |
| % of measures-median+ | | | 95% | 60% | 75% |

√ = within range ____ = score at or above group median
* = beyond upper limit of range
Note: Number in parentheses is the number of subjects reported for range and median.

# APPENDIX N
# RAW DATA FOR STUDENTS EXCLUDED DURING SCREENING

## RAW DATA FOR STUDENTS EXCLUDED DURING SCREENING IN CLASSROOM 1
(with group summary information)

| Student | dig/min<br>#| cntg<br>% | add<br>% | CTBS | B1<br>●/X | B2<br>●/X | B3<br>●/X | B4<br>●/X |
|---|---|---|---|---|---|---|---|---|
| 1A | 20 | 100 | 90 | 32 | 2/10 |  | 0/15 | 0/11 |
| 1B | 54 | 100 | 20 | 1 | 2/29 | 1/23 | 2/33 | 5/32 |
| 1C | 26 | 90 | 100 | 57 | 1/18 | 7/27 | 5/8 | 11/5 |
| 1D | 25 | 100 | 95 | 44 | 2/1 |  | 4/18 | 0/4 |
| Group Median | 47.5 | 100 | 100 | 69 | 3/32 | 2/22.5 | 2/19 | 3/36.5 |

| Student | PRE<br>% | C1<br>%<br>●/X | C2<br>%<br>●/X | C3<br>%<br>●/X | R4<br>%<br>●/X | R5<br>%<br>●/X | R6<br>%<br>●/X | MN7<br>●/X | A8<br>%<br>●/X | A9<br>%<br>●/X | A10<br>%<br>●/X | POST<br>% |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1A | 15 | 80<br>1/1 | 100<br>8/0 | 100<br>5/2 | 100<br>2/0 | 60 90<br>6/0 | 60 100<br>8/0 | 6/7 | 80<br>10/2 | 80<br>9/1 | 90 | 45<br>90 |
| 1B | 10 | 100<br>2/21 | 90<br>0/23 | 80<br>2/53 | 70 100<br>2/11 | 80<br>3/11 | 100 | 3/57 | 80<br>8/0 | 80<br>5/2 | 70 80<br>9/2 | 85<br>90 |
| 1C | 50 | 100<br>6/4 | 100<br>8/4 | 100<br>10/2 | 100<br>8/1 | 100<br>6/1 | 90<br>12/3 | 7/2 | 100<br>15/2 | 100<br>12/2 | 100<br>22/0 | 80<br>100 |
| 1D | 15 | 70 100<br>5/31 | 90<br>5/1 | 80<br>3/0 | 100<br>0/2 | 90 | 100<br>8/0 | 5/0 | 80<br>5/0 | 100<br>4/1 | 100<br>8/1 | 85<br>90 |
| Group Median | 12.5 | 90<br>3/7 | 90<br>4/2 | 90<br>8/1 | 100<br>5/2 | 90<br>7/1.5 | 100<br>9/2.5 | 6.5/1 | 90<br>10/1.5 | 100<br>14/2 | 90<br>16/0 | 80 |

RAW DATA FOR STUDENTS EXCLUDED DURING SCREENING IN CLASSROOM 1--CONTINUED

| Student | 11<br>%<br>●/X | 12<br>%<br>●/X | 13<br>%<br>●/X | 14<br>%<br>●/X | 15<br>%<br>●/X | 16<br>%<br>●/X | 17<br>%<br>●/X | 18<br>%<br>●/X | 19<br>%<br>●/X | 20<br>%<br>●/X | 21<br>% |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1A | 80 | 15/2 | 40<br>19/7 | 70<br>14/6 | 60<br>5/2 | 70<br>15/7 | 90<br>20/4 | 60<br>23/7 | 80<br>20/5 | 80<br>23/4 | 90 |
| 1B | 0<br>1/26 | 4/23 | 70<br>3/21 | 40<br>5/27 | 5/14 | 30<br>4/18 | 40<br>5/22 | | 80<br>5/16 | | |
| 1C | 90<br>11/3 | 100<br>12/1 | 80<br>14/2 | 100<br>24/0 | 80<br>12/1 | 100<br>29/0 | 90<br>22/0 | 100<br>14/4 | 80<br>22/0 | 70<br>23/1 | 100 |
| 1D | 90<br>10/2 | 90<br>8/0 | 100<br>9/2 | 60<br>9/1 | 80<br>9/0 | 70<br>9/1 | 80<br>12/2 | 80<br>10/0 | 90<br>11/0 | 90<br>12/0 | 100 |
| Group Median | 100<br>12/1 | 100<br>12/2 | 80<br>15/1 | 70<br>18/0 | 85<br>8/1 | 75<br>16/0 | 90<br>18/1 | 70<br>13/2 | 90<br>20/0 | 80<br>16/1 | 100 |

RAW DATA FOR STUDENTS EXCLUDED DURING SCREENING IN CLASSROOM 3
(with group summary information)

| Student | dig/min<br>\# | cntg<br>% | add<br>% | CTBS | B1<br>●/X | B2<br>●/X | B3<br>●/X | B4<br>●/X | B5<br>●/X | B6<br>●/X | B7<br>●/X | B8<br>●/X |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3A | 22 | 90 | 100 | 58 | 2/6 | 2/10 | 4/30 | 11/53 | 8/80 | 11/74 | 9/77 | 11/73 |
| 3B | 20 | 100 | 95 | 4 | 1/4 | 1/7 | 0/9 | 0/24 | 0/13 | 0/15 | 0/6 | 0/10 |
| 3C | 25 | 100 | 30 | 5 | 0/8 | 0/18 | 1/3 | 4/8 | 0/6 | 3/24 | 0/10 | 0/23 |
| 3D | 47 | 100 | 65 | 30 | 2/16 | 0/17 | 2/27 | 0/36 | 3/33 | 3/62 | 4/51 | 5/58 |
| Group Median | 50 | 100 | 100 | 69.5 | 4/5.5 | 4/8 | 3.5/12.5 | 4/12 | 3/14 | 6/27 | 5/19 | 7/14 |

| Student | PRE<br>% | C1<br>%<br>●/X | C2<br>%<br>●/X | C3<br>%<br>●/X | R4<br>%<br>●/X | R5<br>%<br>●/X | R6<br>%<br>●/X | MN7<br>●/X | A8<br>%<br>●/X | A9<br>%<br>●/X | A10<br>%<br>●/X | POST<br>% |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3A | 10 | 90<br>16/47 | 100<br>8/1 | 90<br>6/1 | 100<br>5/2 | 100<br>11/1 | 100<br>8/0 | 3/1 | 90<br>2/0 | 100<br>2/0 | 90<br>14/2 | 85 |
| 3B | 5 | 100<br>1/2 | 70<br>4/0 | 70 100<br>5/0 | 90<br>2/0 | 60 100<br>3/2 | 2/9 | 3/0 | 60<br>3/1 | 80<br>1/1 | 70<br>4/2 | 30 40<br>60 80 |
| 3C | 5 | 80 | 40 100 | 50 100<br>3/1 | 100<br>1/4 | 90<br>2/6 | 70 100<br>3/11 | 6/3 | 90<br>5/13 | 90<br>5/10 | 80<br>4/16 | 65<br>90 |
| 3D | 5 | 100<br>0/37 | 90<br>1/7 | 60 100<br>2/30 | 100<br>3/16 | 90<br>7/18 | 70 100<br>6/27 | 4/36 | 70 90<br>6/26 | 100<br>4/15 | 80<br>10/13 | 60<br>90 |
| Group Median | 15 | 90<br>4.5/6 | 100<br>9/2 | 90<br>6/3 | 100<br>3.5/4.5 | 85<br>10/3.5 | 100<br>7.5/2 | 5/0 | 90<br>9.5/4.5 | 100<br>10/3 | 90<br>12/2 | 80 |

RAW DATA FOR STUDENTS EXCLUDED DURING SCREENING IN CLASSROOM 3--CONTINUED

| Student | 11<br>%<br>●/X | 12<br>%<br>●/X | 13<br>%<br>●/X | 14<br>%<br>●/X | 15<br>%<br>●/X | 16<br>%<br>●/X | 17<br>%<br>●/X | 18<br>%<br>●/X | 19<br>%<br>●/X | 20<br>%<br>●/X | 21<br>%<br>●/X |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 3A | 90<br>12/0 | 100<br>7/0 | 100<br>9/0 | 100<br>9/4 | 70<br>18/0 | 100<br>22/0 | 100<br>13/0 | 90<br>18/0 | 90<br>16/0 | 100<br>16/1 | 80<br>9/0 |
| 3B | 60<br>1/1 | 70<br>1/4 | 8/0 | 40<br>5/0 | 30<br>5/0 | 30<br>8/0 | 50<br>1/3 | 40<br>2/7 | 40<br>4/2 | 40<br>6/2 | 10<br>6/0 |
| 3C | 100<br>2/5 | 100<br>6/9 | 80<br>4/1 | 40<br>5/5 | 50<br>12/1 | 70<br>10/4 | 80<br>4/4 | 30<br>6/3 | 80<br>11/2 | 100 | 90<br>7/3 |
| 3D | 100<br>12/0 | 90<br>16/21 | 60<br>8/0 | 80<br>11/9 | 90<br>11/2 | 13/5 | 90<br>5/4 | 70<br>13/11 | 70<br>12/8 | 100<br>14/9 | 90<br>12/4 |
| Group Median | 90<br>12/0 | 100<br>8/2 | 80<br>14/0 | 70<br>7/2 | 70<br>14/0 | 70<br>17/0 | 80<br>10/2 | 70<br>16/2 | 80<br>18/1 | 90<br>12/2 | 90<br>14/1 |

RAW DATA FOR STUDENTS EXCLUDED DURING SCREENING IN CLASSROOM 4
(with group summary information)

| Student | dig/min # | cntg % | add % | CTBS | B1 ●/X | B2 ●/X | B3 ●/X | B4 ●/X |
|---|---|---|---|---|---|---|---|---|
| 4A | 24 | 65 | 75 | 8 | 2/3 | 3/10 | 0/5 | 1/10 |
| 4B | 32 | 70 | 45 | 32 | 2/17 | 5/31 | 3/18 | 4/32 |
| Group Median | 53.5 | 100 | 100 | 65 | 2.5/6.5 | 4/10 | 2.5/9 | 3.5/13 |

| Student | PRE % | C1 % ●/X | C2 % ●/X | C3 % ●/X | R4 % ●/X | R5 % ●/X | R6 % ●/X | MN7 ●/X | A8 % | A9 % ●/X | A10 % ●/X | POST % |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 4A | 20 | 80<br>2/9 | 80<br>3/6 | 60 100<br>4/16 | 90 | 7/8 | 100<br>4/22 | 6/26 | 70 | 70 80<br>10/18 | 90<br>6/20 | 55 |
| 4B | 10 | 80 | 70 80<br>5/27 | 90<br>8/34 | 90<br>13/32 | 70 90<br>9/16 | 60 80<br>10/19 | 8/20 | 60 70<br>80 | 40 70<br>80<br>9/27 | 60 70<br>90<br>15/14 | 70<br>80 |
| Group Median | 20 | 100<br>5/19 | 90<br>8/5 | 90<br>6/11 | 100<br>11/19 | 90<br>9/14 | 90<br>10/11.5 | 7/13 | 90 | 100<br>12.5/13.5 | 100<br>13/11.5 | 90 |

| Student | 11 % ●/X | 12 % ●/X | 13 % ●/X | 14 % ●/X | 15 % ●/X | 16 % ●/X | 17 % ●/X | 18 % ●/X | 19 % ●/X | 20 % ●/X | 21 % |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 4A | 100<br>0/2 | | 60<br>6/9 | 60<br>3/0 | 90<br>7/4 | 90<br>9/1 | 90<br>3/2 | 6/2 | 7/1 | 12/0 | |
| 4B | 90<br>6/1 | 90<br>17/19 | 50<br>12/13 | 60<br>14/17 | | | 100<br>14/10 | 40<br>13/12 | 80<br>14/5 | 90<br>17/7 | 60<br>14/2 |
| Group Median | 100<br>11/1 | 100<br>10/5 | 70<br>13/2 | 75<br>9.5/2 | 100<br>18/0 | 100<br>17/2 | 100<br>9/3.5 | 90<br>14.5/3 | 100<br>17/3.5 | 100<br>17.5/2 | 85<br>20.5/2 |

RAW DATA FOR STUDENTS EXCLUDED DURING SCREENING IN CLASSROOM 6
(with group summary information)

| Student | dig/min # | cntg % | add % | CTBS | B1 ●/X | B2 ●/X | B3 ●/X | B4 ●/X | B5 ●/X | B6 ●/X | B7 ●/X | B8 ●/X |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 6A | 53 | 100 | 65 | 38 | 7/50 | 7/64 | 5/66 | 1/53 | 0/54 | 0/54 | 0/54 | 1/53 |
| Group Median | 62 | 100 | 95 | 67 | 3/30 | 7.5/45.5 | 3/53 | 2/53 | 3/53 | 1/53 | 1/54 | 2/52 |

| Student | PRE % | C1 % ●/X | C2 % ●/X | C3 % ●/X | R4 % ●/X | R5 % ●/X | R6 % ●/X | MN7 ●/X | A8 % ●/X | A9 % ●/X | A10 % ●/X | POST % |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 6A | 0 | 100<br>0/54 | 100<br>3/52 | 100<br>6/4 | 100<br>7/1 | 100<br>5/2 | 90<br>6/1 | 3/4 | 80<br>5/2 | 100<br>8/2 | 100<br>11/2 | 80<br>100 |
| Group Median | 5 | 70<br>3/52 | 100<br>6/39 | 100<br>8/23 | 100<br>7/14 | 100<br>9/8 | 90<br>7/14 | 5/0 | 90<br>11/1 | 90<br>8/1 | 100<br>8.5/0 | 85 |

| Student | 11 % ●/X | 12 % ●/X | 13 % ●/X | 14 % ●/X | 15 % ●/X | 16 % ●/X | 17 % ●/X | 18 % ●/X | 19 % ●/X | 20 % ●/X | 21 % ●/X |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 6A | 100<br>12/0 | 90<br>10/0 | 90<br>12/2 | 60<br>5/2 | 70<br>8/0 | 70<br>8/0 | 70<br>5/2 | 100<br>7/0 | 90<br>13/1 | 90<br>11/1 | 60<br>14/0 |
| Group Median | 90<br>7/0 | 90<br>9/1 | 80<br>9/1 | 70<br>5/0 | 70<br>8/1 | 60<br>8/0 | 90<br>9/0 | 80<br>9/1 | 90<br>14/.5 | 85<br>17/0 | 90<br>14/0 |

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## BIOGRAPHICAL SKETCH

Carolyn Anne Harris was born and raised in Miami, Florida, where she received all of her elementary and secondary education. Throughout the elementary grades, Carolyn was frequently asked to read to younger students and to help her classmates during reading times. In sixth grade, she was asked to be a kindergarten teacher's helper. After that experience, she found herself beginning to think seriously about becoming a teacher.

During high school, Carolyn's interests were focused on science and math. In her junior and senior years, she was the recipient of summer research awards from the National Science Foundation and the Miami Heart Institute. The summer research activities stimulated a keen interest in scientific investigation, and Carolyn entered the University of Florida as an honors student in September of 1965.

By her sophomore year in college, Carolyn's interests were again centered on teaching. Observations at the University of Florida laboratory school helped her decide that she was most interested in early childhood education and the elementary grades.

Carolyn completed her junior and senior years of college at the University of Miami in Coral Gables, Florida. She graduated with a B.Ed. (cum laude) in January of 1969. Carolyn substituted in several elementary schools in South Miami and taught first grade before moving back to Gainesville in 1971.

In 1974, Carolyn received the M.Ed. degree in curriculum and instruction from the University of Florida with certification in the areas of reading and early childhood education. During the period of graduate study, she organized and taught a private kindergarten class.

Following graduation, Carolyn worked for the Alachua County School Board for 5 years as a fifth grade teacher. During that time, Carolyn also taught reading through the Alachua County Adult Education Program.

In the early 1980s, Carolyn worked as a teaching assistant in the Reading Lab at Santa Fe Community College in Gainesville, Florida. Later, she was hired as a resource teacher at a private elementary school. When the school acquired computers, Carolyn took appropriate courses at Santa Fe Community College and designed a curriculum for development of computer literacy.

In 1987, Carolyn entered the doctoral program at the University of Florida in the area of special education with an emphasis in learning disabilities and a minor in reading. During her doctoral program, Carolyn was trained in the Learning Strategies Curriculum at the University of Kansas. She provided strategy training for university students and supervised strategy implementation. Carolyn also assisted with teaching and supervising students in the Precision Teaching course in the Department of Special Education. Carolyn was nominated for the Graduate Student Teaching Award by the Department of Special Education for two consecutive years.

Following completion of the doctoral program, Carolyn is interested in further work as a trainer in the Learning Strategies Curriculum for preservice and inservice teachers. Carolyn is also interested in serving as an educational consultant for implementation of effective teaching practices in mainstream education.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Cecil D. Mercer

Cecil D. Mercer, Chairman
Professor of Special Education

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Vivian I. Correa

Vivian I. Correa
Professor of Special Education

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Henry T. Fillmer

Henry T. Fillmer
Professor of Instruction and
Curriculum

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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This dissertation was submitted to the Graduate Faculty of the College of Education and to the Graduate School and was accepted as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

December, 1992

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